Smooth Transitions from High School to Community College

MATHEMATICS CURRICULUM ALIGNMENT GUIDE

A Gulf Coast PASS Project
Smooth Transitions from High School to Community College

Mathematics Curriculum Alignment Guide

A Gulf Coast PASS Project

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HOUSTON ENDOWMENT

A PHILANTHROPY ENDOWED BY JESSE H. AND MARY GIBBS JONES

Generous funding from the Houston Endowment to the University of Texas made this Gulf Coast region initiative possible.

STUDENT SUCCESS INITIATIVES
THE UNIVERSITY OF TEXAS AT AUSTIN

The University of Texas at Austin’s Student Success Initiative provided direction, coordination, and evaluation.

HOUSTON A+ CHALLENGE

Houston A+ Challenge led the college awareness focus and convened three annual GC PASS Institutes.

IEBC

INSTITUTE for EVIDENCE-BASED CHANGE
Informing Decisions - Improving Practice - Increasing Student Success

IEBC led data collection and analysis, developed web-based tools, facilitated curriculum alignment teams, and oversaw production of this guide.
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INTRODUCTION

THE GULF COAST PARTNERSHIP FOR ACHIEVING STUDENT SUCCESS PROJECT

This Mathematics Curriculum Alignment Guide (and its counterpart, the English Curriculum Alignment Guide) is the culmination of three years of ongoing collaboration between eight community colleges and 11 independent school districts (ISDs) in the Texas Gulf Coast region. The project was funded by a grant from The Houston Endowment to The University of Texas at Austin’s Student Success Initiatives to support the Gulf Coast Partnership for Achieving Student Success (GC PASS) initiative, the primary goals of which are to:

- increase college readiness among high school graduates,
- ease student transitions between high school and community college, and
- increase student success in community college developmental courses.

Other partners included Houston A+ Challenge, which led the college awareness focus of the project and convened three annual GC PASS Institutes to ensure that project participants were informed of the latest national, state, and local research as well as promising practices. The Institute for Evidence-Based Change (IEBC) led the facilitation of aligning curricula between high school and higher education by creating and facilitating vertical alignment teams, collecting and analyzing data, developing web-based tools, designing and integrating achievement metrics, and overseeing the production of these guides.

BACKGROUND

The community colleges and K-12 districts involved in GC PASS collectively serve more than 720,000 Texas students each year. All GC PASS community colleges previously spent five or more years as participants in Achieving the Dream (AtD, http://achievingthedream.org/), a nationwide effort to increase the rate at which students—especially students of color and low-income—achieve, persist, and complete community college. Each college’s GC PASS project built on its work from AtD by setting targets to reduce the need for remediation, accelerate student progress through developmental courses, and increase student success in “gatekeeper” courses.

All K-12 public school districts in GC PASS were invited into the initiative by their respective community colleges that worked alongside them to foster a college-going culture. Outcomes targeted at the K-12 level included alignment of curricula and student
expectations; expanded offerings of dual enrollment, including remedial courses and college credit courses; and increased student and parent understanding of (and successful applications for) financial aid. Participating ISDs and community colleges, including more than 50 instructors, in this partnership were:

<table>
<thead>
<tr>
<th>College</th>
<th>Public K-12 School District (ISD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazosport College</td>
<td>Brazosport ISD</td>
</tr>
<tr>
<td>College of the Mainland</td>
<td>Hitchcock ISD</td>
</tr>
<tr>
<td>Galveston College</td>
<td>Galveston ISD</td>
</tr>
<tr>
<td>Houston Community College System</td>
<td>Houston ISD and Spring Branch ISD</td>
</tr>
<tr>
<td>Lee College</td>
<td>Goose Creek ISD</td>
</tr>
<tr>
<td>Lone Star College System</td>
<td>Cypress-Fairbanks ISD and Spring ISD</td>
</tr>
<tr>
<td>San Jacinto College District</td>
<td>Pasadena ISD and Sheldon ISD</td>
</tr>
<tr>
<td>Wharton County Junior College</td>
<td>Wharton ISD</td>
</tr>
</tbody>
</table>

These faculty teams were charged with exploring and clarifying mathematics alignment in the Gulf Coast region with respect to three key gateway courses in high school and college: (high school) Algebra II, College Algebra, and (college) Precalculus.

**TEXAS PATHWAYS**

**Mathematics Alignment in Texas**

Success in Algebra II was recently designated by the Texas legislature as the appropriate indicator of student preparedness for College Algebra. The alignment analysis in this guide agrees overwhelmingly with this recommendation. While Algebra I and Geometry stop just short of covering many skills critical for success in College Algebra, Algebra II overlaps College Algebra by a considerable margin. Students who have mastered Algebra II at the high school level are not far from having mastered College Algebra, lacking only depth on most topics and leaving just a handful of skills entirely unaddressed. This overlap appears in the guide in some redundancy between the Algebra II exit-level expectations and the College Algebra entry-level expectations. (See pg. 8, “Development of Assessments,” for more information about Aligned Course Expectations, or ACEs.) Additionally, the attentive reader will notice that many of the College Algebra entrance-level skills are subsumed by Algebra II exit-level skills.
Many conversations around the appropriateness of this margin have taken place throughout this alignment work. More questions have been raised than have been answered, such as:

- Is such an overlap appropriate?
- Should College Algebra be shifted to encompass more Precalculus skills and fewer Algebra II skills?
- Would student success suffer significantly due to a reduction in this margin?
- Is College Algebra even necessary for students who have completed Algebra II?

This guide will only allow the reader to understand the state of mathematics alignment in the Gulf Coast region; the answers to the questions it raises are left to the reader.

Multiple Pathways

Recent work in the state seems to indicate that the days when all degree-seeking college students take College Algebra are coming to an end. Less algebraically intensive pathways are emerging at institutions across the state. For all but a minority of science, technology, engineering, and mathematics (STEM) students, the college mathematics emphasis is shifting from hard algebra skills to critical thinking, decision-making, and quantitative literacy. These significant changes do not undermine the importance of the alignment questions raised above. While high schools are increasingly diverting future non-STEM majors onto paths that do not pass through Algebra II, Algebra II will continue to support the success of future STEM majors who need to complete College Algebra and Precalculus.

NEW CORE CURRICULUM GUIDELINES IN TEXAS—FALL 2014

The Texas Higher Education Coordinating Board recently adopted curriculum guidelines that specify minimum learning outcomes each academic course must include, no matter the college/university that is offering the course in the state of Texas. The guidelines must be enacted by fall semester 2014. Individual institutions can add additional learning outcomes to courses, if appropriate, and not all of the general education outcomes are assessed in every course. (See the Texas Higher Education Academic Course Guide Manual at http://www.thecb.state.tx.us/reports/pdf/2969.pdf?CFID=15967412&CFTOKEN=16469097 for specific details for each course.) It is recommended that teachers from all grade levels familiarize themselves with these guidelines and refer to them when developing lesson plans.
INTRODUCTION

The new guidelines are:

• critical thinking skills: to include creative thinking, innovation, inquiry, and analysis, evaluation, and synthesis of information;

• communication skills: to include effective development, interpretation, and expression of ideas through written, oral, and visual communication;

• empirical and quantitative skills: to include the manipulation and analysis of numerical data or observable facts resulting in informed conclusions;

• teamwork: to include the ability to consider different points of view and to work effectively with others to support a shared purpose or goal;

• personal responsibility: to include the ability to connect choices, actions, and consequences to ethical decision-making; and

• social responsibility: to include intercultural competence, knowledge of civic responsibility, and the ability to engage effectively in regional, national, and global communities.

PROCESS: THE WORK OF ALIGNMENT

This math curriculum alignment work was created by a subgroup of the GC PASS consortium called Math Professional Alignment Councils (MPACs). The MPACs were made up of more than 50 representative math faculty (see Appendix C) from each of the community colleges and high school districts involved in the project. These teams were charged with clarifying and strengthening mathematics alignment in the Gulf Coast region from 11th grade through the first year of community college. Broad alignment of curricula such as this should allow students to transition seamlessly from any of the involved high schools to any of the involved community colleges in the region. For example, a student from Hitchcock High School should be prepared at such a level that she can attend Lonestar College (or any community college on the Gulf Coast) and achieve success in her first college-level mathematics class without remediation.

Thus, this guide aims to provide faculty and administrators in secondary and higher education in the Gulf Coast region with a better understanding of:

• the scope of math curricula for high school Algebra II, College Algebra, and Precalculus;

• the alignment between these three courses; and,

• how recurring mathematics content and skills evolve in depth and rigor as students move from course to course.
INTRODUCTION

THE GC PASS COLLABORATIVE PROCESS

This project got its start in 2012 when more than 50 math instructors from Gulf Coast high schools and community colleges gathered in Houston to address mathematics curricular expectations. From the outset, there was passionate questioning, back-and-forth sharing, and lively discussion. Both high school and community college faculty clearly wanted to prepare their students for success and therefore wanted to understand their faculty counterparts and the structures in which they worked. Collaborating on something as crucial as aligning curricula to ensure student success is, in a word, messy.

Fortunately, there was already a proven method for this type of collaboration, developed by the Institute for Evidence-Based Change (IEBC) in the organization’s curriculum alignment work in several states across the U.S. Trusting in this process is what made the 3-year GC PASS Curriculum Alignment Project successful. Instructors were divided into eight MPACs according to their community college–high school partnership region (see “Math Professional Alignment Council,” Appendix C). MPACs were given monthly assignments to accomplish, and then the larger group of MPAC leads gathered for all-day convenings every quarter to review, critique, and engage in Socratic dialogue around individual MPAC work.

Using information from a course gap analysis completed by each MPAC, faculty developed three types of assessments: a computational problem, a conceptual problem, and a real world application. (See “How to Use this Guide” on page 10 for specifics).

DEVELOPMENT OF ASSESSMENTS

Assessment samples were written for each entry-level and exit-level ACE in order to specify the appropriate depth and rigor of course concepts. Assessment samples for each ACE were written collaboratively by secondary and higher education faculty. Each category of assessment samples underwent a rigorous revision process by other professional alignment councils. The final collection was reviewed and agreed upon by the partnerships.

Assessments were then developed to indicate level of rigor that should be expected. Assessments were assigned a taxonomy, indicating rigor according to Daggett’s Rigor/Relevance Framework (see figure on page 9).
INTRODUCTION

We hope that these illustrative assignments support mathematics instructors at both the high school and college levels to provide their students with the skills and knowledge necessary to smoothly transition from high school to college without need for remediation. Curriculum alignment necessitates deliberative practice, repeated conversations among all stakeholders, and a tolerance for trial and error. Fortunately, these are all capacities among mathematics teachers, whose problem-solving expertise engenders in them the ability and interest to take up this work. Therefore, we hope that this guide will be used to inspire others to embark on the kinds of collaborative projects that created it.

Shelly Valdez, Project Co-Director

Katheryn Horton, Project Co-Director

Matt Lewis, Project Liaison
HOW TO USE THIS GUIDE

PURPOSE OF THIS GUIDE

This guide was created by Gulf Coast regional high school and community college faculty to demonstrate ways in which mathematics curricula for Algebra II and College Algebra or Precalculus can be aligned to provide students with increasing depth and rigor as they transition from one grade level to the next in high school and then from high school to college. Broad alignment of curricula such as this should allow students to transition seamlessly and achieve success in their first college-level mathematics class without remediation.

FORMAT OF THE GUIDE

Aligned Course Expectations

The guide is organized according to assessments of the Aligned Course Expectations (ACEs) that were identified by the Math Professional Alignment Council (MPAC). The ACEs (listed in full in Appendix B) are broken into four major collections:

- Exit-Level Algebra II
- Entry-Level College Algebra
- Exit-Level College Algebra, and
- Entry-Level Precalculus

Each entry-level ACE represents a foundational skill necessary for success in the associated course. Each exit-level ACE represents a skill a student must master in order to satisfy the associated course expectations (see Appendix B).

For ease of reference, each major collection of ACEs is divided into seven categories, as summarized in the table on page 11. These categories align with the Algebra II categories in the Texas Essential Knowledge and Skills (TEKS), which specify the K-12 curriculum standards in Texas (http://ritter.tea.state.tx.us/rules/tac/chapter111/index.html).
Most ACEs have three assessment samples: a computational problem, a conceptual problem, and a real world application. Assessments are laid out in the following format:

**Aligned Course Expectation**

A Computational Assessment

- Level of rigor (listed as “Taxonomy” based on Daggett’s Rigor/Relevance Framework, p. 9)
- The computational assessment answer

A Conceptual Assessment

- Level of rigor (listed as “Taxonomy” based on Daggett’s Rigor/Relevance Framework, p. 9)
- The conceptual assessment answer

An Application (Real Life) Assessment

- Level of rigor (listed as “Taxonomy” based on Daggett’s Rigor/Relevance Framework, p. 9)
- The application assessment answer

**Development of the ACEs**

The Aligned Course Expectations (ACEs) reflect the consensus of all eight alignment teams working on the GC PASS Initiative. Representatives from all partnerships gathered at regional meetings to establish guidelines for identifying, writing, and revising all entry-level and exit-level ACEs. ACEs to be written were divided into categories and assigned to MPACs, which were made up of secondary and higher education faculty who then worked together to write each collection of ACEs. The resulting body of ACEs was then analyzed and revised at regional meetings of all partnerships at the end of the 2013 academic year.

<table>
<thead>
<tr>
<th>Category</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Functions &amp; Miscellany</td>
</tr>
<tr>
<td>B</td>
<td>Systems</td>
</tr>
<tr>
<td>C</td>
<td>Exponential &amp; Logarithmic Expressions &amp; Equations</td>
</tr>
<tr>
<td>D</td>
<td>Rational Expressions &amp; Equations</td>
</tr>
<tr>
<td>E</td>
<td>Radical Expressions &amp; Equations</td>
</tr>
<tr>
<td>F</td>
<td>Polynomial Expressions &amp; Equations</td>
</tr>
<tr>
<td>G</td>
<td>Conic Sections</td>
</tr>
</tbody>
</table>
HOW TO USE THIS GUIDE

The scope of high school Algebra II and College Algebra are captured in collections of exit-level expectations that present the content as a collection of measurable skills. The exit-level expectations specify the key content in each course, allowing the reader to understand how each of these courses manifests in the Gulf Coast region. In particular, the exit-level expectations for College Algebra represent the essential core upon which all eight community colleges could agree.

The MPACs also developed entry-level ACEs for College Algebra and Precalculus to facilitate the exploration of alignment with high school Algebra II. These entry-level ACEs indicate those foundational skills that are critical to success. Comparing exit-level Algebra II ACEs to the corresponding entry-level College Algebra or Precalculus ACEs elucidates any gaps or overlaps existing between the courses. For ease of comparison, corresponding ACEs across various levels are grouped together in multicourse alignment tables (see Appendix A).

OTHER USES FOR THIS GUIDE

The guide might also be used as a tool for teacher professional development on high school and college campuses. It could serve to enhance discussion among colleagues about the complexity of alignment issues, the development of new courses, or the creation of a new program.
ASSESSMENTS FOR EXIT-LEVEL ALGEBRA II

A. Functions & Miscellany .................................................................................................................... 16
B. Systems .................................................................................................................................................... 43
C. Exponential & Logarithmic Expressions & Equations ................................................................. 62
D. Rational Expressions & Equations ................................................................................................. 81
E. Radical Expressions & Equations .................................................................................................... 102
F. Polynomial Expressions & Equations ............................................................................................ 106
G. Conic Sections .................................................................................................................................... 133
FUNCTIONS & MISCELLANY

• ALIGNED COURSE EXPECTATIONS A01

Transform between representations of functions (e.g., table of values, equation, inequality, graph, verbal description).

COMPUTATIONAL ASSESSMENT

Taxonomy: C

Determine a relationship between the $x$ and $y$ values. Write an equation.

\[
\begin{array}{c|cccc}
 x & 1 & 2 & 3 & 4 \\
 y & 4 & 5 & 6 & 7 \\
\end{array}
\]

Computational Assessment Answer

$y = x + 3$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Graph the equation determined by the relationship between the $x$ and $y$ values.

\[
\begin{array}{c|cccc}
 x & 1 & 2 & 3 & 4 \\
 y & 4 & 5 & 6 & 7 \\
\end{array}
\]
Conceptual Assessment Answer

Graph of $y = x + 3$

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

A video club costs $25 to join. Each video that is rented costs $2.50. Let $v$ represent the number of videos. Identify the independent and dependent variables. Then, write a rule in function notation for the situation.

Application (Real Life) Assessment Answer

Independent: number of videos rented
Dependent: total cost; $f(v) = 2.5v + 25$
ALIGNED COURSE EXPECTATIONS A02

Evaluate functions at a given input value.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Given \( g(x) = 5x + 3 \), find \( g(5) \).

Computational Assessment Answer:

\[ g(5) = 5(5) + 3 = 28 \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Use the graph of \( g(x) \) to find the value of \( g(3) \).

Computational Assessment Answer

\[ g(3) = 8 \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A computer manufacturing company calculates their profit in thousands of dollars using the function \( P(x) \) where \( x \) is thousands of computers sold. The function is defined as \( P(x) = 15x - 8x + 2 \).

Find the profit for selling 38,000 computers.

Application (Real Life) Assessment Answer

\[ P(38) = 15(38) - 8(38) + 2 = 268 \]

The company will make $268,000 profit for selling 38,000 computers.
ALIGNED COURSE EXPECTATION A03

Identify the domain and range of a function from various representations, including table of values, equation, inequality, graph, and verbal description.

COMPUTATIONAL ASSESSMENT

Taxonomy: C

State the domain and range of the given graph:

Computational Assessment Answer

Domain: all real numbers.

Range: all real numbers greater than –3.
CONCEPTUAL ASSESSMENT

Taxonomy: C

Sketch the graph of $-2x + 4y = 4$ and state the domain and range.

Conceptual Assessment Answer

Domain and range are both all real numbers.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

Brian has 64 flowers for a big party decoration. In addition, he is planning to buy some flower arrangements that have 18 flowers each. All of the arrangements cost the same. Brian is not sure yet about the number of flower arrangements he wants to buy, but he has enough money to buy up to 5 of them. Write a function rule to describe how many flowers Brian can buy. Let $x$ represents the number of flower arrangements Brian buys. Find a reasonable range for the function.

Application (Real Life) Assessment Answer

Brian has enough money to buy only up to 5 flower arrangements, so he can buy 0, 1, 2, 3, 4, or 5 flower arrangements. These are the only reasonable values for the domain.

Substitute these values into the function rule to find the range values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$18 \times 0 + 64 = 64$</td>
</tr>
<tr>
<td>1</td>
<td>$18 \times 1 + 64 = 82$</td>
</tr>
<tr>
<td>2</td>
<td>$18 \times 2 + 64 = 100$</td>
</tr>
<tr>
<td>3</td>
<td>$18 \times 3 + 64 = 118$</td>
</tr>
<tr>
<td>4</td>
<td>$18 \times 4 + 64 = 136$</td>
</tr>
<tr>
<td>5</td>
<td>$18 \times 5 + 64 = 154$</td>
</tr>
</tbody>
</table>
**Aligned Course Expectation A04**

Identify an appropriate function to fit given data.

**Computational Assessment**

**Taxonomy: A**

Complete the table shown; then, determine whether the table shows a linear relationship.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$5.75 + (x + x + x) / 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>?</td>
<td>17</td>
</tr>
<tr>
<td>?</td>
<td>20</td>
</tr>
</tbody>
</table>

**Computational Assessment Answer**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$5.75 + (x + x + x) / 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

The table shows a linear function.
CONCEPTUAL ASSESSMENT

Taxonomy: C

Tell whether the relation is a direct variation. Explain.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-10</th>
<th>-9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>20</td>
<td>18</td>
<td>-2</td>
</tr>
</tbody>
</table>

**Conceptual Assessment Answer**

This is a direct variation because it can be written as $y = kx$ where $k = -2$.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

Thomas is a car salesman. He is paid a salary of $1,600 per month plus $300 for each car that he sells. His salary can be modeled by the equation $f(x) = 300x + 1600$ where $x$ is the number of cars sold. Graph this equation and state the domain and range.

**Application (Real Life) Assessment Answer**

Domain: all real numbers greater than or equal to zero.

Range: all real numbers greater than or equal to 1600.
ALIGNED COURSE EXPECTATION A05

Identify the reasonable domain and range for a situation modeled by a function, including continuous and discrete situations.

COMPUTATIONAL ASSESSMENT

No assessment.

CONCEPTUAL ASSESSMENT

Taxonomy: C

A ball is launched at 75.5 feet per second from the top of a building that is 28 feet tall. The height in feet of the ball, \( t \), seconds after it is launched is given by \( f(t) = -16t^2 + 75.5t + 28 \). What is the practical domain for the function \( f(t) \)?

Conceptual Assessment Answer

The practical domain for \( f(t) \) is from the time the ball is launched, \( t = 0 \) seconds, to the time the ball hits the ground, \( t \approx 5.06 \) seconds.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

The cost of postage is 49¢ for the first ounce, plus 23¢ for each ounce after that. State the reasonable domain and range for this function.

Application (Real Life) Assessment Answer

Domain will be all weights greater than 0. Range will be .49, .49 + .23, .49 + .23 + .23, etc.
ALIGNED COURSE EXPECTATION A06

Identify the type of parent function presented given unique characteristics of that parent function.

COMPUTATIONAL ASSESSMENT

No assessment.

CONCEPTUAL ASSESSMENT

Taxonomy: C

Match the given graphs to their parent function equations:

1)
2)

3)
a) \( f(x) = x^3 + 75.5t \)

b) \( f(x) = |x| \)

c) \( f(x) = \sqrt{x} \)

**Conceptual Assessment Answer**

Graph 1 is function (b); graph 2 is function (a); graph 3 is function (c).

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: B**

The population growth of turkeys in a specific prairie region can be modeled using an exponential function. The initial population of turkeys is 100. Sketch a possible graph to represent the function.

**Application (Real Life) Assessment Answer**

Answers may vary, but it should be clear that the y-intercept is at 100. Below is a possible graph.
**ALIGNED COURSE EXPECTATION A07**

Graph parent functions, including square, absolute value, square root, cube root, cubic, reciprocal, logarithmic, and exponential functions.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: B**

Write the equation $y = -2x^2 + 8x - 5$ in vertex form. Identify the vertex and axis of symmetry.

**Computational Assessment Answer**

Vertex form: $y = -2(x - 2)^2 + 3$

Vertex: $(2, 3)$

Axis of symmetry: $x = 2$

**CONCEPTUAL ASSESSMENT**

**Taxonomy: B**

Given the equation $y = 2|x + 4| - 1$, list the transformations to the parent function and graph the function.
Conceptual Assessment Answer

Graph is translated left 4 units and down 1 unit, vertically stretched by a factor of 2.

Application (Real Life) Assessment

Taxonomy: C

The flight of a golf ball when hit from a tee can be modeled by the equation

\[ y = 0.25x - 0.001x^2 \]

where \( x \) represents the horizontal distance of the ball (in yards) and \( y \) represents the height of the ball (in yards). What is the maximum height of the golf ball? How far away from the tee does the golf ball land?

Application (Real Life) Assessment Answer

Height is 15.6 yards; distance is 250 yards.
**ALIGNED COURSE EXPECTATION A08**

Graph various transformations of parent functions, including linear, quadratic, exponential, square root, absolute value, logarithmic, etc.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: B

Graph the function \( f(x) = -2x \).

**Computational Assessment Answer**

![Graph of the function \( f(x) = -2x \)](image-url)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Graph the equation $y = 2\sqrt{x} + 2 - 3$.

Conceptual Assessment Answer
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C

Write an equation for the graph.

A speed camera is set 5 feet from a nearby road and switched on at \( t = 0 \) seconds. A bike traveling along the road at a constant speed of 4 feet per second passes the camera exactly 2 seconds later. The graph below shows a graph of the distance, \( d(t) \), between the camera and the bike as a function of time, \( t \). Find an equation for the distance, \( d(t) \), as a function of time, \( t \).

Application (Real Life) Assessment Answer

\[ d = 4|t - 2| + 5 \text{ where } t \geq 0 \]
ALIGNED COURSE EXPECTATION A09
Transform between zeros and x-intercepts of functions.

COMPUTATIONAL ASSESSMENT
Taxonomy: A
The function $f(x)$ has zeros at $x = 2, x = 3, x = 5$. State the x-intercepts.

Computational Assessment Answer
$x$-intercepts: $(2, 0), (-3, 0), (5, 0)$

CONCEPTUAL ASSESSMENT
Taxonomy: C
State the zeros of the given graph:

Conceptual Assessment Answer
$(2, 0)$ and $(4, 0)$
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C

The height of an object dropped from 200 feet at $t$ seconds is given by the function $h(t) = -16t^2 + 324$. The function is graphed below. Find the approximate time when the object will hit the ground.

Application (Real Life) Assessment Answer

The object will hit the ground after about 4.5 seconds.
**ALIGNED COURSE EXPECTATION A10**

Simplify an algebraic expression using the rules of exponents.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Simplify \( \frac{4x^2y^3}{6x^3y} \).

**Computational Assessment Answer**

\( \frac{2}{3}x^3y^2 \)

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Write an expression that makes the following statement true:

\( \frac{12x^5y^2}{?} = 2x^2y^3 \)

**Conceptual Assessment Answer**

\( 6x^3y^{-1} \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C

Write an equation for the volume of the cylinder shown below in terms of $x$.

Application (Real Life) Assessment Answer

$$\frac{\pi x^5}{12}$$
**ALIGNED COURSE EXPECTATION A11**

Use symbols to represent unknowns and variables in problem situations.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Use symbols to represent the following statement: a number plus $5$.

**Computational Assessment Answer**

$x + 5$

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

The length of a rectangular window is expressed as $5x$. The width of the window is expressed as $3x + 2$. Write the area of the window in terms of $x$.

**Conceptual Assessment Answer**

\[ A = (5x)(3x + 2) = 15x^2 + 10x \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

Joey is purchasing topsoil and mulch for his landscaping company. He knows that a bag of mulch weighs 23 pounds and a bag of topsoil weighs 40 pounds. He does not know how many bags of each he will buy. Write a mathematical expression Joey can use to represent the total weight of his purchase.

Application (Real Life) Assessment Answer

Let $x$ represent the number of bags of mulch, so $23x$ is the total weight of mulch. Let $y$ represent the number of bags of topsoil, so $40y$ is the total weight of top soil. Thus, $23x + 40y$ will give the total weight of both products.
**ALIGNED COURSE EXPECTATION A12**

Describe functional relationships for given problem situations by writing equations or inequalities.

**COMPUTATIONAL ASSESSMENT**

No assessment.

**CONCEPTUAL ASSESSMENT**

No assessment.

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: C**

The cost to manufacture bottles of water is 50¢ per bottle. The cost to ship each bottle is 25¢. Write a function, C, for the cost of manufacturing and shipping x bottles.

**Application (Real Life) Assessment Answer**

\[ C(x) = 0.5x + 0.25x = 0.75x \]
ALIGNED COURSE EXPECTATION A13

Simplify and perform operations on rational expressions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Perform the indicated operation:

\[
\frac{2}{x} + \frac{3}{x+5}.
\]

Computational Assessment Answer

\[
\frac{2}{x} + \frac{3}{x+5} = \frac{5x+10}{x(x+5)}.
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Break the given rational expression into a sum of rational terms:

\[
\frac{2x^2 + 3x + 3}{x^2 + x}.
\]

Conceptual Assessment Answer

\[
\frac{2x^2}{x^2 + x} + \frac{3x}{x^2 + x} + \frac{3}{x^2 + x}.
\]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

The annual cost of running a washing machine is given by the expression

\[
\frac{570 + 97x}{x}
\]

The annual cost of running a dryer is given by the expression

\[
\frac{650 + 102x}{x}
\]

Find a single expression for the total annual cost for using both machines.

**Application (Real Life) Assessment Answer**

\[
\frac{570 + 97x}{x} + \frac{650 + 102x}{x} = \frac{1220 + 199x}{x}
\]
SYSTEMS

● ALIGNED COURSE EXPECTATION B01

Solve a system of two equations in two variables using graphs.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following system using the graphing method:

\[ 3x - y = 5 \]
\[ 2x - 3y = -6 \]

Computational Assessment Answer

Solution: (3, 4)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Samantha was asked to solve the following system of equations by the graphing method:

\[ y = -x + 3 \]
\[ x + y = -\frac{2}{3} \]

Samantha’s graph and solution are shown below:

Samantha stated that “there are an infinite number of solutions to the system.”

State whether Samantha’s work and solution are correct. If there are any errors in the work or solution, correct them and give the correct solution.
Conceptual Assessment Answer

Samantha’s graph is correct, but her solution is incorrect. She should have stated that the system has no solution.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Jeremy wanted to rent a pair of skates. He could rent them from Bob’s Skate Shop for $5 plus $2 for each hour that he uses them. At Sally’s Roller Rage, he could rent skates for $2 plus $3 for each hour of use. After how many hours of use would Jeremy pay the same amount with either plan? How much would it cost Jeremy for that amount of time with either plan? Solve the problem by graphing, and answer the question in a complete sentence.

Application (Real Life) Assessment Answer

The cost of the two options would be the same after 3 hours. It would cost Jeremy a total of $11 for 3 hours with either plan.
ALIGNED COURSE EXPECTATION B02

Solve a system of two equations in two variables using a table.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use a table of values to solve the following system:

\[ y = 4x - 3 \]
\[ y = \frac{3}{2}x + 2 \]

Computational Assessment Answer

The solution is \((2, 5)\).

CONCEPTUAL ASSESSMENT

Taxonomy: C

Describe how you would determine the solution of the system below by using the table, which was created for the system below.

\[ 12x - 5y = -7 \]
\[ 4x + 2y = 5 \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y_1)</th>
<th>(y_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.5</td>
<td>.2</td>
<td>3.5</td>
</tr>
<tr>
<td>-.25</td>
<td>.8</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1.4</td>
<td>2.5</td>
</tr>
<tr>
<td>.25</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>.5</td>
<td>2.6</td>
<td>1.5</td>
</tr>
<tr>
<td>.75</td>
<td>3.2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3.8</td>
<td>.5</td>
</tr>
</tbody>
</table>
**Conceptual Assessment Answer**

Look for a matching value in columns $Y_1$ and $Y_2$, which would represent the $y$ value of the solution. Then, look in the first column for the $x$ value of the solution. The solution is $(.25, 2)$.

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: D**

The Westlake Village Transit Authority has two options for bus riders on route 1. The first option is to purchase a Smooth Pass Card for $45 and then pay only 50¢ per ride. The second option is to simply pay $2 per ride and not purchase a card. After how many rides will the cost of the two options be the same? Enter the appropriate equations in your calculator, and use the table in your calculator to determine the answer.

**Application (Real Life) Assessment Answer**

The cost of the two options would be the same after 30 rides.
**ALIGNED COURSE EXPECTATION B03**

Solve a system of two equations in two variables using substitution and elimination.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the solution to the system below by the substitution method:

\[ \begin{align*}
    x - y &= 4 \\
    \frac{3}{7}x + \frac{2}{3}y &= 5
\end{align*} \]

**Computational Assessment Answer**

The solution is \((7, 3)\).

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Sam wanted to solve the following system by the elimination method:

\[ \begin{align*}
    8x + 6y &= -2 \\
    10x - 9y &= -8
\end{align*} \]

Find the mistake in Sam’s work, and explain what he should have done to obtain the correct solution. Finish by finding the correct solution.
Sam's work:

\[3(8x + 6y) = -2\]
\[2(10x - 9y) = -8\]

\[24x + 18y = -2\]
\[20x - 18y = -8\]

\[44x = -10\]
\[x = -\frac{10}{44}\]
\[x = -\frac{5}{22}\]

\[10\left(-\frac{5}{22}\right) - 9y = -8\]
\[-\frac{25}{11} - 9y = -8\]
\[-9y = \frac{63}{11}\]
\[y = \frac{7}{11}\]

Sam's solution is \((-\frac{5}{22}, \frac{7}{11})\).

**Conceptual Assessment Answer**

Sam made a mistake when he multiplied. He should have multiplied every term in the first equations by 3 and every term in the second equation by 2, not just the terms on the left sides of the equations. The correct solution is \((-\frac{1}{2}, \frac{1}{3})\).

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: D**

The local musical sold a total of 298 tickets. The upper level tickets cost $38 and the lower level tickets cost $60. The musical had a total of $16,296 in sales. How many upper level tickets were sold, and how many lower level tickets were sold?

**Application (Real Life) Assessment Answer**

There were 72 upper level tickets sold and 226 lower level tickets sold.
• ALIGNED COURSE EXPECTATION B04

Solve a system of two equations in two variables using matrices.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use an inverse matrix to solve the linear system:

\[
\begin{align*}
5x + 2y &= 14 \\
-x - 4y &= 26
\end{align*}
\]

Computational Assessment Answer

The solution is \((6, -8)\).

CONCEPTUAL ASSESSMENT

Taxonomy: C

What is the inverse matrix that would be used to solve this system?

\[
\begin{align*}
-2x - 9y &= -2 \\
4x + 16y &= 8
\end{align*}
\]

Conceptual Assessment Answer

\[
\begin{pmatrix}
4 & 9 \\
-1 & -\frac{1}{2}
\end{pmatrix}
\]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The University Interscholastic League team often ordered food from the Best Kitchen Restaurant when they were practicing for a meet. One week they ordered four cheese pizzas, three ham sandwiches, and five Greek salads. The food cost $96.25. The next week they ordered two cheese pizzas, five ham sandwiches, and five Greek salads, and the food came to $88.75. The third week of practice they ordered six cheese pizzas, three ham sandwiches, and three Greek salads, totaling $101.25. All totals were before any taxes were added. Use an inverse matrix to determine the unit price of a pizza, a ham sandwich, and a Greek salad.

Application (Real Life) Assessment Answer

One pizza cost $10, one ham sandwich cost $6.25, and one Greek salad cost $7.50.
**ALIGNED COURSE EXPECTATION B05**

Create a system of two equations in two variables to represent a problem situation.

**COMPUTATIONAL ASSESSMENT**

*Taxonomy: A*

If the sum of two integers is 26 and five times the lowest number is equal to the highest number minus 98, which of the following equations could not be used as one of the systems of equations for the given statement? (Use x as the smallest number and y as the largest number.)

a) \( x + y = 26 \)

b) \( 5x - y = -98 \)

c) \( 5x = y - 98 \)

d) \( y = 5x + 98 \)

**Computational Assessment Answer**

b) \( 5x - y = -98 \)

**CONCEPTUAL ASSESSMENT**

*Taxonomy: C*

Sierra was asked to create a system of equations for the following word problem, and her answer is stated below. Please state if Sierra was correct or incorrect and explain your answer:

Jake’s age is four years less than two times his younger sister Allison’s age. If the sum of Jake’s age and Allison’s age is 44, set up a system of equations for this scenario using \( a \) for Allison’s age and \( j \) for Jake’s age.

Sierra’s answer:

\[
\begin{align*}
    j &= 4 - 2a \\
    j + a &= 44
\end{align*}
\]
Conceptual Assessment Answer

Sierra’s first equation is wrong because it is not saying, “four less than two times the age of Allison,” which is what the problem stated. Sierra is actually saying, “four subtract two times the age of Allison.” If this had been the equation, it wouldn’t have had a solution and the top equation would have had a negative age somewhere, which we know can’t be true.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Three families went out to eat at Burger Palace. The first family bought five burgers, three fries, and five drinks for a total of $55.25. The second family purchased three burgers, three fries, and three drinks for $38.25. The last family ordered two burgers, two fries, and one drink for $22. Set up a $3 \times 3$ system of equations. You need to define the variable that you are using as well.

Application (Real Life) Assessment Answer

\[ b = \text{burgers} \]
\[ f = \text{fries} \]
\[ d = \text{drinks} \]
\[ 5b + 3f + 5d = 55.25 \]
\[ 3b + 3f + 3d = 38.25 \]
\[ 2b + 2f + d = 22 \]
ALIGNED COURSE EXPECTATION B06

Solve a real world application system involving a linear system.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following system of equations:

\[ 6x + 6y = 7 \]
\[ 2x - 27y = -17 \]

Computational Assessment Answer:

\[ x = \frac{1}{2} \]
\[ y = \frac{2}{3} \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

The sum of two numbers is 375 and four times the smaller number is equal to double the sum of the bigger number and 105. Steve says the two numbers are 215 and 160. Mary says the two numbers are 250 and 125. Which one of these two students is correct? Explain your answer.

Conceptual Assessment Answer

Steve is correct, although answers may vary. If you plug Mary’s answer into the system of equations, the numbers come out false on the second equation.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A pizza from Mario’s Magic Mozzarella costs $7.50 plus 80¢ per topping. A pizza from Luigi’s Lasagna and More costs $9 plus 50¢ per topping. How many toppings would you have to purchase in order to make Luigi’s Lasagna and Mario’s Magic Mozzarella equivalent in price, and what would that price be?

Application (Real Life) Assessment Answer

Five toppings; $11.50
ALIGNED COURSE EXPECTATION B07

Solve a system of three linear equations in three variables.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following system of equations for $x$, $y$, and $z$:

\[
\begin{align*}
2x + 4y + 3z &= -14 \\
-2y &= 6x - 4z \\
-2x - 4y - 8z &= 34
\end{align*}
\]

Computational Assessment Answer

\[
\begin{align*}
x &= -3 \\
y &= 1 \\
z &= -4
\end{align*}
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Josh has worked out the following $3 \times 3$ system of equations but is struggling to explain each step. Write out the starting matrix, explain Josh's first three steps, and include the augmented solution matrix.

\[
\begin{align*}
6x + 3y + 3z &= 42 \\
-x + y - 3z &= 15 \\
2x - 3y - z &= -14
\end{align*}
\]
### Conceptual Assessment Answer

Answers will vary; here is a sample answer:

Original matrix:

\[
\begin{bmatrix}
0 & 0 & -26 & -156 \\
-1 & 0 & -10 & -62 \\
0 & -1 & -7 & -46
\end{bmatrix}
\]

Step 1: Josh multiplied the first row by \( \frac{1}{3} \) in order to reproduce a new row 1: \( \left( \frac{1}{3} r_1 \rightarrow r_1 \right) \).

Step 2: Josh then added row 1 and two times row 2 in order to create a new row 1: \( (r_1 + 2r_2 \rightarrow r_1) \).

He also added row 3 and two times row 2 in order to create a new row 3: \( (r_3 + 2r_2 \rightarrow r_3) \).

Step 3: Josh then added row 1 to three times row 3 to produce a new row 1: \( (r_1 + 3r_3 \rightarrow r_1) \).

Augmented solution matrix:

\[
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 4 \\
0 & 0 & 1 & 6
\end{bmatrix}
\]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Three families went out to eat at Burger Palace. The first family bought five burgers, three fries, and five drinks for a total of $55.25. The second family purchased three burgers, three fries, and three drinks for $38.25. The last family ordered two burgers, two fries, and one drink for $22. Set up and solve a $3 \times 3$ system of equations using $b$ for burgers, $f$ for fries, and $d$ for drinks.

Application (Real Life) Assessment Answer

\[ 5b + 3f + 5d = 55.25 \]
\[ 3b + 3f + 3d = 38.25 \]
\[ 2b + 2f + d = 22 \]

\[ b = 5 \]
\[ f = 4.25 \]
\[ d = 3.50 \]
ALIGNED COURSE EXPECTATION B08:

Distinguish between inconsistent, dependent, and independent systems of linear equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Classify the following system of equations correctly with the correct explanation:

\[ y = 3x - 4 \]
\[ 9x - 3y = -12 \]

a) Inconsistent. They are parallel lines because both have a slope of 3 and different \( y \)-intercepts.

b) Dependent. They are the exact same line because the slope is 3 and the \( y \)-intercept is 4, indicating that they lie on top of each other.

c) Independent. There is a definite solution for the system of equations, which is (3, 5).

d) Dependent. There is a definite solution for the system of equations, which is (3, 5).

Computational Assessment Answer

a) Inconsistent. They are parallel lines because both have a slope of 3, and they have different \( y \)-intercepts.
CONCEPTUAL ASSESSMENT

Taxonomy: C

Classify each of the following three systems of equations as either an inconsistent, dependent, or independent system of equations:

a) \[3x + 5y = 19\]
   \[12x + 20y = 1\]

b) \[5x + 3y = 4\]
   \[2x + y = 3\]

   \[2x + 6y = 12\]
   \[12y = -4x + 24\]

Conceptual Assessment Answer

a) Independent solution: (5, -7)

b) Inconsistent. This is a set of parallel lines. Both of these lines have a slope of \(-\frac{3}{5}\) but different \(y\)-intercepts.

c) Dependent: Each of the lines when moved to slope intercept form is \(y = -\frac{1}{3}x + 2\).

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

For the given scenario, write a system of equations and determine if the system is dependent, inconsistent, or independent. Briefly explain your reason for each classification.

At the Groverton Grocery Store they are having a sale on zucchini and squash. The zucchini cost 79¢ per pound and the squash cost 99¢ per pound. Suppose Shelby went into the store and bought 12 pounds of just zucchini and squash and paid a total of $11.28. Let \(x\) represent pounds of zucchini and let \(y\) represent pounds of squash.
Application (Real Life) Assessment Answer

\[ x + y = 12 \]
\[ .79x + .99y = 11.28 \]

This system is independent because there would be a definite solution, which would be 4 pounds of zucchini and 8 pounds of squash.
**EXPONENTIAL & LOGARITHMIC EXPRESSIONS & EQUATIONS**

● **ALIGNED COURSE EXPECTATION C01**

Evaluate logarithmic expressions by applying properties of logarithms.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

1. Evaluate each of the following expressions:
   a) \( \log_{10} \frac{1}{1000} \)  
   b) \( \log_2 8 \)  
   c) \( \log_5 \frac{1}{5} \)  
   d) \( \ln e^6 \)

2. Simplify \( \log_{1000} \frac{1}{1000} + \ln \frac{e^{-2}}{e} \).

**Computational Assessment Answer**

1. a) \(-3\); b) \(3\); c) \(-1\); d) \(6\)
2. \(-6\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Use rules of logarithms to simplify the following expression:

\( \frac{\log a}{\log a^2} - \log 10b + \log b \).
Conceptual Assessment Answer

\[
\frac{-2}{3}
\]

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

In chemistry, pH is given by the formula

\[
pH = -\log[H^+],
\]

where \( H^+ \) is the hydrogen ion concentration in moles per liter. What is the pH of sour milk with a hydrogen ion concentration of 0.00004? Round the answer to three decimal places.

Application (Real Life) Assessment Answer

4.398
**ALIGNED COURSE EXPECTATION C02**

Identify the domain and range for an exponential function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Give the domain and range for each exponential function:

a) \( f(x) = e^{x-2} \)

b) \( f(x) = e^x - 2 \)

c) \( f(x) = 2 - e^{x-2} \)

**Computational Assessment Answer**

a) Domain: \((-\infty, \infty)\) or \(x\) can be any real number; Range: \((0, \infty)\) or \(y > 0\).

b) Domain: \((-\infty, \infty)\) or \(x\) can be any real number; Range: \((-2, \infty)\) or \(y > -2\).

c) Domain: \((-\infty, \infty)\) or \(x\) can be any real number; Range: \((-\infty, 2)\) or \(y > 2\).

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

The bacteria in a petri dish double every three minutes. If the petri dish is full of bacteria after 40 minutes, how long did it take before the petri dish was one-quarter full?
Conceptual Assessment Answer

The petri dish was half full after 37 minutes and one-quarter full after 33 minutes.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Let \( A \) represent the mass of Fermium-253 (253\text{Fe}) (in grams), whose half-life is 3 days. The quantity of Fermium-253 present after \( t \)-days is

\[
A = 4 \times \left( \frac{1}{2} \right)^{t/3}.
\]

a) Determine the initial quantity (when \( t = 0 \)).

b) Determine the quantity present after nine days.

c) Determine the range of the function that corresponds to the domain.

Application (Real Life) Assessment Answer

a) 4g

b) 0.5g

c) \([0.25, 4]\)
ALIGNED COURSE EXPECTATION C03

Solve exponential equations using algebraic methods.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve each of the following exponential equations. If rounding is required, give the solution rounded to four decimal places.

a) $4^{2x} = 16$

b) $3e^x = 10$

Computational Assessment Answer

a) $x = 1$

b) $x = 1.2040$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Two basic methods used to solve exponential equations are (1) to write each side of the equation using the same base exponential, or (2) to convert the equation from exponential form to logarithm form. When would it be best to use the first method? When would it be best to use the second method? Give examples to support your answer.

Conceptual Assessment Answer

Answers will vary; here is a sample answer: If the exponential equation can be rewritten using the same base on each side of the equation, then the equation can be solved without logarithms.
Example of exponential equation that can be solved without logs:

\[ 5^{2x-1} = 1 \]
\[ 5^{2x-1} = 5^0 \]
\[ 2x - 1 = 0 \]
\[ x = \frac{1}{2} \]

Example of exponential equation that cannot be solved without logs:

\[ 5^{2x-1} = 2 \]
\[ 2x - 1 = \log_5 2 \]
\[ 2x - 1 = \frac{\log 2}{\log 5} \]
\[ 2x - 1 = 0.4307 \]
\[ x = 0.7153 \]

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: D**

a) John deposits $700 in a bank that offers a rate of interest of 2.5%, compounded quarterly. Calculate the amount of money John has in the bank after 5 years.

b) Amy deposits $700 in another bank that offers interest compounded annually. Her investment doubles in 20 years. Find the rate of interest that her bank is offering.

**Application (Real Life) Assessment Answer**

a) $792.90

b) 3.53%
ALIGNED COURSE EXPECTATION C04

Write an exponential model given key characteristics (i.e., base, rate of growth/decay, data points, etc.).

COMPUTATIONAL ASSESSMENT

Taxonomy: A

A town’s population doubles in size every 20 years. If the population is currently 20,000, what is the equation that will give the population \( x \) years from now?

Computational Assessment Answer

\[ y = 20,000 \times 2^{\frac{x}{20}} \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Write an exponential model whose graph passes through the point \((0, a)\) whose value, \(y\), doubles with every increase in \(x\) of 5 units.

Conceptual Assessment Answer

\[ y = a \times 2^{\frac{x}{5}} \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Bacteria in a culture are growing exponentially with time, as shown in the table below.

Bacteria Growth

<table>
<thead>
<tr>
<th>Hour</th>
<th>Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>1</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>1800</td>
</tr>
</tbody>
</table>

Which of the following equations expresses the number of bacteria, $y$, present at any time, $t$?

a) $y = 200 + 3^t$

b) $y = 200(3^t)$

c) $y = 3^t$

d) $y = 600(3^t)$

Application (Real Life) Assessment Answer

b) $y = 200(3^t)$
ALIGNED COURSE EXPECTATION C05

Identify a reasonable domain and range for a situation modeled by an exponential function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

The demand curve for a new granola bar can be represented by \( D=1200(0.95)^p \), where \( p \) represents the price per bar in cents. Find the domain and range for this function.

Computational Assessment Answer

Domain: \([0, \infty)\); Range: \([1200, \infty)\)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Why does the range of an exponential function only include all the positive real numbers and no negative numbers?

Conceptual Assessment Answer

Because the graph of an exponential function never intersects or goes below the x-axis, which means the y-values (range) are never zero or negative. This is due to the property of exponents that says, given any positive number raised to any power, it will never produce a negative number or zero.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The population of a town in Smithville can be modeled by the function \( P = 30,000 + 1.024^x \), where \( x \) is the number of years after 2003 and \( P \) is the population. What are the domain and range?

Application (Real Life) Assessment Answer

Domain: \([0, \infty)\); Range: \([30000, \infty)\)
**ALIGNED COURSE EXPECTATION C06**

Determine the inverse relationship between exponential and logarithmic equations using algebraic methods, graphing, and tables.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Determine if the following functions are inverses of each other:

\[ f(x) = \ln x \]
\[ g(x) = e^x \]

**Computational Assessment Answer**

Yes

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Estimates of the amounts (in billions of dollars) of U.S. online advertising spending from 2007 through 2011 are shown in the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Advertising Spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>21.1</td>
</tr>
<tr>
<td>2008</td>
<td>23.6</td>
</tr>
<tr>
<td>2009</td>
<td>25.7</td>
</tr>
<tr>
<td>2010</td>
<td>28.5</td>
</tr>
<tr>
<td>2011</td>
<td>32.0</td>
</tr>
</tbody>
</table>
An exponential growth model that approximates this data is \( S = 10.33e^{0.1022t}, \quad 7 < t < 11 \)
where \( S \) is the amount of spending (in billions) and \( t = 7 \) represents 2007. If \( S^{-1} \) exists, what does it represent in the context of this problem?

**Conceptual Assessment Answer**

\( S^{-1} \) represents the time (in years) after 2000 at which a given spending level was attained.

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: D**

Biologists use the logarithmic model \( n = k \log(A) \) to estimate the number of species, \( n \), that live in a region of area, \( A \). In this model, \( k \) represents a constant. Find the inverse of this logarithmic model. Use words to describe what the inverse represents.

**Application (Real Life) Assessment Answer**

The inverse is \( A = 10^{n/k} \).

This formula will give the region of area, \( A \), in which \( n \) species lives.
ALIGNED COURSE EXPECTATION C07

Transform an exponential equation into logarithmic form and vice versa.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Rewrite each equation into an equivalent exponential equation.

a) \( \log_2 8 = 3 \)

b) \( 4 = \ln x \)

c) \( 4 + \log x = y \)

Computational Assessment Answer

a) \( 2^3 = 8 \)

b) \( e^4 = x \)

c) \( 10^{y-4} = x \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Let \( y = f(x) \), where \( f(x) \) is a basic logarithm whose graph has a vertical asymptote at \( x = 5 \); then, determine the domain and asymptote of the graph of \( y = f^{-1}(x) \).

Conceptual Assessment Answer

Domain: \( (-\infty, \infty) \)

Asymptote: \( y = 5 \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The population of Dallas $n$ years after 1990 may be approximated by the exponential function $P = 1,006,831(1.011)^n$. A student stated this formula is equivalent to the formula

$$n = \log_{1.011} \frac{P}{1,006,831}.$$ 

Is this true?

Application (Real Life) Assessment Answer

Yes.
**ALIGNED COURSE EXPECTATION C08**

Simplify exponential expressions by applying laws of exponents.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Simplify the exponential expressions using the laws of exponents:

a) \(5^0\)

b) \(3^2\)

c) \(x^2 \cdot x^3\)

d) \(\frac{k^5}{k^3}\)

e) \((ab)^3\)

f) \((3^3)^4\)

g) \(\left(\frac{2}{3}\right)^4\)
Computational Assessment Answer

a) 1
b) \( \frac{1}{3^2} \)
c) \( x^5 \)
d) \( k^2 \)
e) \( a^3b^3 \)
f) 3^8
g) \( \frac{2^4}{3^4} \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Use a graphing calculator to graph \( f(x) = 2^x \) and \( g(x) = 5^x \) on the same set of axes.

a) Examine very large positive values of \( x \) on the graphs of \( f(x) \) and \( g(x) \). As the values of \( x \) increase, what happens to the values of \( f(x) \) and \( g(x) \)?

b) Examine very large negative values of \( x \) on the graphs of \( f(x) \) and \( g(x) \). As the values of \( x \) decrease, what happens to the values of \( f(x) \) and \( g(x) \)?

Conceptual Assessment Answer

a) The values of \( f(x) \) and \( g(x) \) increase rapidly without bound.

b) The values of \( f(x) \) and \( g(x) \) get closer and closer to 0.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

An asteroid travels at a speed of $5^8$ miles per day. How many miles will it travel in $5^3$ days?

Application (Real Life) Assessment Answer

$5^{13}$ miles
ALIGNED COURSE EXPECTATION C09

Identify a reasonable domain and range for a situation modeled by a logarithmic function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A
An amoeba divides into two amoebas every hour. You can represent the number $N$ of amoebas after $t$ hours by the formula $t = \log_2 N$. Find the domain and range.

Computational Assessment Answer

Domain: $(0, \infty)$; Range: $(-\infty, \infty)$

CONCEPTUAL ASSESSMENT

Taxonomy: C
The brightness of two stars can be compared using their magnitude difference, $d$, or brightness ratio, $r$. The relationship between $d$ and $r$ can be approximated by $d = 2.5 \log r$. Explain why negative numbers are excluded from the domain of this logarithmic function.

Conceptual Assessment Answer

Logarithmic functions are only defined on numbers that are strictly bigger than zero. This is because the logarithmic function is a different way of writing exponents. This is due to the property of exponents that says given any positive number raised to any power, it will never produce a negative number or zero.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The percentage of adult height attained by a girl who is \( x \) years old can be modeled by \( f(x) = 35\log(x) \), where \( x \) represents the girl’s age (from 5 to 15) and \( f(x) \) represents the percentage of her adult height. Use a calculator to determine the domain and range of this function. Round your answer to four decimal places.

Application (Real Life) Assessment Answer

Domain: [5, 15]; Range: [56.3303, 94.7818]
RATIONAL EXPRESSIONS & EQUATIONS

**ALIGNED COURSE EXPECTATION D01**

Identify the attributes of a given rational function (asymptotes, intercepts, discontinuities, end behavior) from its graph and/or equation.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A
Determine any points of discontinuity for the rational function

\[ y = \frac{2x^2 + 5}{x^2 - 2x}. \]

Computational Assessment Answer

\[ x = 0, x = 2 \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: C
Describe what happens to the graph of

\[ y = \frac{3x + 5}{x - 2} \]

as the absolute value of \( x \) increases.

Conceptual Assessment Answer

The points on the graph will get closer to the horizontal line \( y = 3 \).
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Kyla and Sam are starting a business making necklaces. There is an original development cost of $1,200 and they can create each necklace for $2.35. A function that models the average cost, $y$, for selling $x$ necklaces is

$$y = \frac{2.35x + 1200}{x}.$$ 

What is the horizontal asymptote of this function, and what does it mean for the situation?

Application (Real Life) Assessment Answer

The horizontal asymptote $y = 2.35$ means that the average cost will never go below $2.35 since that is how much each necklace costs to produce.
ALIGNED COURSE EXPECTATION D02

Determine the equation of a rational function of the form \( f(x) = \frac{1}{x} \) or \( f(x) = \frac{1}{x^2} \) from its graph using transformations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Determine the equation of the rational function in the following graph:

![Graph of a rational function](image)

Computational Assessment Answer

\[ f(x) = \frac{1}{x - 3} + 2. \]
CONCEPTUAL ASSESSMENT

Taxonomy: C

Susan wrote the equation for the function seen in the graph below. Her function was

\[ f(x) = \frac{1}{(x-2)^2}. \]

Describe the mistake Susan made and write the correct equation.

Conceptual Assessment Answer

Susan wrote the equation for a function translated two units to the right instead of two units to the left. The correct equation for the function is

\[ f(x) = \frac{1}{(x+2)^2}. \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Richie Rich and some of his friends plan to charter a private jet to see the World Cup in Brazil. The cost of the charter will be $20,000, but they can split that cost equally among themselves. In addition, each passenger will have to pay a $4,500 fee for food, beverage, and insurance. The graph below shows the total cost per passenger as a function of the number of passengers actually going on the flight.

If the flat rate of the charter increases to $25,000 but the per-passenger fee is reduced so that if 16 passengers were on the flight, the rate for each passenger would be the same as with the old rate, how will the graph change to reflect these new rates?

Application (Real Life) Assessment Answer

The graph will have a vertical stretch of 25,000 from the parent function

$$y = \frac{1}{x},$$

as opposed to a vertical stretch of 20,000, and the horizontal asymptote will be at $y=4,187.5$ instead of at $y=4,500$. 
**ALIGNED COURSE EXPECTATION D03**

Convert a rational function from a ratio of two polynomials to a polynomial plus remainder and vice versa.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Convert the rational function

\[
y = \frac{2x^2 + 6x}{x^2 - 9}
\]

into a form containing a polynomial plus a remainder.

**Computational Assessment Answer**

\[
y = 2 + \frac{6x + 18}{x^2 - 9}.
\]

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Mary wanted to convert

\[
y = 3 + \frac{1}{x - 2}
\]

to a ratio of two polynomials. What would her first step be?
Conceptual Assessment Answer

Create a common denominator by multiplying 3 by the fraction

\[
\frac{(x - 2)}{(x - 2)}
\]

The result is

\[
y = \frac{3(x - 2) + 1}{x - 2}.
\]

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The cost for Amanda’s birthday party is $125 to rent the party room and $10 for each person’s meal. Rewrite the function

\[
C(x) = \frac{125 + 10x}{x}
\]

as a polynomial plus a remainder.

Application (Real Life) Assessment Answer

\[
C(x) = 10 + \frac{125}{x}
\]
ALIGNED COURSE EXPECTATION D04

Graph a rational function from its equation in the form

\[ f(x) = \frac{1}{x} \text{ or } f(x) = \frac{1}{x^2}, \]

including functions with transformations.

Computational Assessment

Taxonomy: A

Graph the function

\[ y = \frac{-2}{(x + 1)^2} + 3. \]

Computational Assessment Answer
**CONCEPTUAL ASSESSMENT**

Taxonomy: C

If \( f(x) = \frac{1}{x} \) or \( f(x) = \frac{1}{x^2} \), and \( g(x) = 3f(x-2) \), then write the equation of \( g(x) \) in terms of \( x \) and describe how to transform \( f(x) \) to create the graph of \( g(x) \).

**Conceptual Assessment Answer**

\[ g(x) = \frac{3}{x-2} \]

Begin with the graph of \( f(x) = \frac{1}{x} \), expand vertically by 3, and shift right 2 units.

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

A pharmaceutical company has calculated that a child’s dosage \( y = 1 - \frac{1}{x} \) is dependent upon her age in years, \( x \). Draw a graph representing the dosage.
Application (Real Life) Assessment Answer
ALIGNED COURSE EXPECTATION D05

Evaluate rational functions given specific inputs/outputs.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

For the function \( g(x) = \frac{x^2 - 9}{2x + 6} \), find \( g(-1) \).

Computational Assessment Answer

\( g(-1) = -2 \)
CONCEPTUAL ASSESSMENT

Taxonomy: B

\[ y = \frac{x + 1}{x^2 - 5x + 6}, \]

Giada noticed that when she input 2 for \( x \) in the function
the output value was undefined. Explain.

Conceptual Assessment Answer

When \( x \) is 2, the denominator of the fraction is zero, making the fraction undefined.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Brittney made 16 of her last 24 basketball free throw attempts, a success percentage of

\[ y = \frac{16 + x}{24 + x} \]

about 67%. The function

can be used to model her free throw success rate for \( x \) more consecutive successful free throw attempts. Approximately how many consecutive free throws must she now make in order to raise her success rate to 75%?

Application (Real Life) Assessment Answer

About 8
ALIGNED COURSE EXPECTATION D06

Identify the domain and range for a rational function in interval notation and inequalities.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

What is the range of

\[ f(x) = \frac{1}{(4x + 1)^2} + 7? \]

Computational Assessment Answer

\[ \{ y : y > 7 \} \]

CONCEPTUAL ASSESSMENT

Taxonomy: B

What is the domain and range of the function shown below?

Conceptual Assessment Answer

Domain: \[ \{ x : x < -3 \text{ or } x > -3 \} \]

Range: \[ \{ y : y < 6 \text{ or } y > 6 \} \]

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
ALIGNED COURSE EXPECTATION D07

Identify a reasonable domain and range for a situation modeled by a rational function.

COMPUTATIONAL ASSESSMENT

Taxonomy: D

We can now build houses made out of concrete using a giant 3-D printer. The printer places layer over layer of concrete to create each of the walls. A smaller version of this printer used to create scale models costs $24,000. The material to build a scale model of a home costs $300. The function that represents the average cost per scale model is

\[ C = \frac{300m + 24,000}{m}. \]

What are the domain and range for this function?

Computational Assessment Answer

Domain: \((0, \infty)\)

Range: \((300, \infty)\)

CONCEPTUAL ASSESSMENT

Taxonomy: D

A company sells oatmeal in containers that hold 500 cubic centimeters, at the most. They use the function

\[ h = \frac{500}{\pi r^2} \]

to determine the height and radius of the container. What is a reasonable domain of the function so that the height of the container is between 7 cm. and 25 cm. tall?
Conceptual Assessment Answer

The radius should be between about 2.5 cm. and 4.8 cm.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

You are deciding whether or not it is worth it to join a new gym based on what you will spend overall in a year. It costs $100 to join and then $59 monthly. What is a reasonable range for cost per month if you will be a member from 1 to 2 years?

Application (Real Life) Assessment Answer

$63.17 to $67.33
ALIGNED COURSE EXPECTATION D08

Solve rational inequalities.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the inequality \( \frac{x}{x-3} \geq 5 \).

Computational Assessment Answer

3 < x ≤ 3.75

CONCEPTUAL ASSESSMENT

Taxonomy: C

Jeff says the solution to the inequality \( \frac{1}{x-1} \geq 1 \) is the set [1, 2]. Is Jeff correct?

Conceptual Assessment Answer

No, the solution set should not include 1. The correct answer is (1, 2).
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

To simulate the mass that can be supported by a cantilever, a math teacher bundled several sticks of dried spaghetti and extended the spaghetti bundle cantilever over the edge of a tabletop. By attaching different weights to the end of this cantilever, the strength of the cantilever could be measured by recording the amount of deflection or bend that occurred in the cantilever. The more pieces of spaghetti bundled together, the stronger the cantilever became. The function below shows the relationship between $D$, the amount of deflection in the cantilever as measured in centimeters, and $n$, the number of spaghetti sticks in the bundle when a 20 gram load is attached to the end of the cantilever. If the amount of deflection that occurs is to stay between 2 and 5 cm., how many sticks of spaghetti should be used to support the load?

$$D = \frac{9.085}{n}$$

Application (Real Life) Assessment Answer

Two, three, or four sticks of spaghetti should be used.
**ALIGNED COURSE EXPECTATION D09**

Solve rational equations using algebraic methods.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the equation \( \frac{x}{5x+5} = \frac{-3}{2x+2} + \frac{2x-3}{x+1} \).

**Computational Assessment Answer**

\[ x = \frac{5}{2} \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Jim was told to solve the equation

\[ \frac{2}{x+3} - \frac{1}{x-1} = \frac{3x}{x^2 + 2x - 3} \]

His work is shown below.

\[
\begin{align*}
\frac{2}{x+3} - \frac{1}{x-1} &= \frac{3x}{(x+3)(x-1)} \\
2(x-1) - (x-1) &= 3x \\
2x - 2 - x + 1 &= 3x \\
x - 1 &= 3x \\
-1 &= 2x \\
\frac{1}{2} &= x
\end{align*}
\]

There is an error in Jim’s work. What mistake did he make?
Conceptual Assessment Answer

Jim did not distribute correctly, as shown in the second term.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Maria is training for a biathlon. She must run a distance of 5 kilometers and cycle for 60 kilometers. If she can cycle three times faster than she can run, and she completes the race in 1.2 hours, then what is her cycling speed?

Application (Real Life) Assessment Answer

62.5 km./hr.
**ALIGNED COURSE EXPECTATION D10**

Use inverse variations (i.e., \( k = xy \)) to solve application problems, such as Boyle’s Law.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: B

Chemists use the equation \( PV = k \) to determine the amount of volume, \( V \), that a gas occupies when under a specific pressure, \( P \), where \( P \) is the pressure in atmospheric pressure (atm) and \( V \) is volume in liters. If a gas has a pressure of 1.35 atm and occupies a volume of 8.20 L, then what pressure will it be under if it occupies a space of 3.21 L?

**Computational Assessment Answer**

3.45 atm.

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

A group of students has been asked to create an explanation for the formula for density

\[
\text{Density} = \frac{m}{V}
\]

to share with the class. Which explanation is correct?

a) The density of an object is directly proportional to the volume and the mass of the object.

b) The mass of an object is directly proportional to the volume of the object.

c) The density of the object is indirectly proportional to the volume of the object.

d) The volume of the object has no proportional relationship with the density or mass of an object.

**Conceptual Assessment Answer**

c) The density of the object is indirectly proportional to the volume of the object.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A balloon with a capacity of 1 liter is filled with helium at 1.3 atm. (atmospheric pressure). If the balloon is then squeezed so that its volume is reduced by 25%, what pressure is the helium experiencing? Use the formula $PV = k$, where $P$ is the pressure in atm. and $V$ is volume in liters.

Application (Real Life) Assessment Answer

1.73 atm.
RADICAL EXPRESSIONS & EQUATIONS

**ALIGNED COURSE EXPECTATION E01**
Find the inverse of a square root function using algebraic methods.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the inverse of the function \( f(x) = \sqrt{2x - 1} \).

Computational Assessment Answer

\[ f^{-1}(x) = \frac{1}{2}x^2 + \frac{1}{2}, \text{ where } x \geq 0. \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Determine the domain and range of \( f(x) = \sqrt{x + 2} \); then, find the inverse of \( f(x) \) as well as the domain and range of the inverse.

Conceptual Assessment Answer

Domain of \( f(x) \): \([-2, \infty)\)

Range of \( f(x) \): \([0, \infty)\)

Domain of \( f^{-1}(x) \): \([0, \infty)\)

Range of \( f^{-1}(x) \): \([-2, \infty)\)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The function \( v(d) = \sqrt{19.62d} \) gives the instantaneous velocity in feet per second of a freely falling object that has traveled distance \( d \) feet. Find \( d(v) \) and the inverse of \( v(d) \) and describe in practice terms the meanings of \( d(19.81) = 20 \).

Application (Real Life) Assessment Answer

\[ d(v) = \frac{v^2}{19.62} \]

When the object is traveling at a velocity of 19.81 ft./sec., the object will have fallen 20 ft.
ALIGNED COURSE EXPECTATION E02

Solve square root equations and inequalities using algebraic methods and recognize extraneous solutions.

COMPETATIONAL ASSESSMENT

Taxonomy: A

Solve the following radical equation for \( x \): \( \sqrt{x + 5} = x - 1 \).

Computational Assessment Answer

\( x = 4 \) (\( x = -1 \) is extraneous)

CONCEPTUAL ASSESSMENT

Taxonomy: B

How many real solutions are there to \( \sqrt{x + c} = -2 \)?

Conceptual Assessment Answer

This equation has no solution. The left-hand side is a positive quantity, while the right-hand side is a negative quantity.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The function \( v(d) = \sqrt{19.62d} \) gives the instantaneous velocity in feet per second of a freely falling object that has traveled distance \( d \) feet. How far must a freely falling object fall to reach a velocity of 12 feet/second? (Round your answer to the nearest tenth of a foot.)

Application (Real Life) Assessment Answer

The object must fall 7.3 feet.
**ALIGNED COURSE EXPECTATION E03**

Identify the domain and range of a square root function and express them in interval notation and in terms of inequalities.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Determine the domain and range of the radical function \( f(x) = \sqrt{15-3x} - 2 \).

**Computational Assessment Answer**

Domain: \((-\infty, 5]\)

Range: \([-2, \infty)\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Consider the function \( f(x) = \sqrt{x} \). Given that a real number, \( a \), is in the domain of \( f(x) = \sqrt{x} \), what can be said about \( a \)?

**Conceptual Assessment Answer**

\( a \geq 0 \)

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

The distance to the horizon, \( d \), that you can see from a given height, \( h \), is given by the equation \( d = 1.2 \sqrt{h} \). Find the domain of \( d \) as a function of \( h \).

**Application (Real Life) Assessment Answer**

\( h \geq 0 \)
POLYNOMIAL EXPRESSIONS & EQUATIONS

• ALIGNED COURSE EXPECTATION F01

Write a quadratic function given key characteristics (i.e., vertex, direction, points on the parabola, or roots).

COMPUTATIONAL ASSESSMENT

Taxonomy: B
Suppose an arrow is fired straight up into the air at an initial speed of 32 feet/second from an initial height of 8 feet. Write a quadratic function, \( h(t) \), which gives the arrow’s height, \( h \), as a function of time, \( t \).

Computational Assessment Answer

\[ h(t) = -16t^2 + 32t + 8 \]

CONCEPTUAL ASSESSMENT

Taxonomy: D
Suppose an arrow is fired straight up into the air at an initial speed of \( V_0 \) feet/second from an initial height of \( h_0 \) feet. Write a quadratic function, \( h(t) \), which gives the arrow’s height, \( h \), as a function of time, \( t \).

Conceptual Assessment Answer

\[ h(t) = -16t^2 + v_0 t + h_0 \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose an arrow is fired straight up into the air at an initial speed of 32 feet/second from an initial height of 48 feet. After how many seconds, t, does the arrow strike the ground?

Application (Real Life) Assessment Answer

\[ t = 3 \]
ALIGNED COURSE EXPECTATION F02

Describe a quadratic function using multiple representations, such as a table of values, equation, graph, or a verbal description.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Consider the quadratic function given by \( f(x) = -x^2 - 4x + 3 \). Complete the following tabular representation of this functional relationship:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Computational Assessment Answer

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>
CONCEPTUAL ASSESSMENT

Consider the quadratic function given by $f(x) = -x^2 - 4x + 3$. Complete the pairs in the following tabular representation of this function:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$-a^2 - 4a + 3$</td>
</tr>
<tr>
<td>$a+1$</td>
<td></td>
</tr>
<tr>
<td>$a+2$</td>
<td></td>
</tr>
</tbody>
</table>

Conceptual Assessment Answer

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$-a^2 - 4a + 3$</td>
</tr>
<tr>
<td>$a+1$</td>
<td>$-a^2 - 6a - 2$</td>
</tr>
<tr>
<td>$a+2$</td>
<td>$-a^2 - 8a - 9$</td>
</tr>
</tbody>
</table>
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A bowling ball was dropped from a height of 144 feet and timed as it fell. The following table summarizes the ball's height (in feet), $h$, at various times (in seconds), $t$:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$h(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>144</td>
</tr>
<tr>
<td>1</td>
<td>128</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Given that the ball's height (in feet), $h$, at time (in seconds) $t$ is given by a function of the form $h(t) = at^2 + c$, use the table above to find $a$ and $c$.

Application (Real Life) Assessment Answer

$a = -16$ and $c = 144$
**ALIGNED COURSE EXPECTATION F03**

Find the domain and range of quadratic functions.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the domain and range of \( f(x) = x^2 - 4x + 5 \).

**Computational Assessment Answer**

Domain: \((-\infty, \infty)\)

Range: \([1, \infty)\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Consider the graph of the function \( f(x) \) below. Use the graph to determine the domain and range of the function.
Conceptual Assessment Answer

Domain: \((-\infty, \infty)\)

Range: \([1, \infty)\)

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose a firework is fired straight up from the ground. Its height is given by

\[ h(t) = -4.9t^2 + 98t. \]

What is the reasonable domain of this problem?

Application (Real Life) Assessment Answer

Reasonable domain: \([0, 20]\)
**ALIGNED COURSE EXPECTATION F04**

Transform one form of a quadratic equation (vertex form, standard form) into the other form.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Let \( f(x) = 2(x + 1)^2 - 4 \). Express \( f(x) \) in the form \( f(x) = ax^2 + bx + c \).

**Computational Assessment Answer**

\[ f(x) = 2x^2 + 4x - 2 \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Suppose \( f(x) = -3(x + 2)^2 + c \) and \( f(x) = -3x^2 - 12x - 8 \) represent the same quadratic function. Find \( c \).

**Conceptual Assessment Answer**

\[ c = 4 \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose \( f(x) = -3(x + 2)^2 + c \). How large should \( c \) be so that the \( y \)-intercept of the graph of \( f(x) \) is positive?

Application (Real Life) Assessment Answer

\( c > 12 \)
• ALIGNED COURSE EXPECTATION F05

Create a model of a real world situation by applying quadratic functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: B

A rectangular garden is to be built from 80 feet of fencing. Let \( x \) be the width of the garden. Write a quadratic function, \( A(x) \), that gives the area of the garden (in square feet) as a function of the width, \( x \).

Computational Assessment Answer

\[ A(x) = -x^2 + 40x \]

CONCEPTUAL ASSESSMENT

Taxonomy: D

A rectangular garden is to be built from 80 feet of fencing. Let \( x \) be the width of the garden. If \( A(x) = -x^2 + 40x \) gives the area of the garden (in square feet) as a function of its width, \( x \), will this garden have a maximum possible area?

Conceptual Assessment Answer

The garden will have a maximum area since \( A(x) = -x^2 + 40x \) has a maximum value.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A rectangular garden is to be built from 80 feet of fencing. Let $x$ be the width of the garden. Write a quadratic function, $A(x)$, that gives the area of the garden (in square feet) as a function of the width, $x$, and find the maximum area.

Application (Real Life) Assessment Answer

$A(x) = -x^2 + 40x$

Width = 20 feet

Maximum area = 400 square feet
Aligned Course Expectation F06

Identify the domain and range for a situation modeled by a quadratic function.

Computational Assessment

Taxonomy: B

Suppose a firework is fired straight up from the ground. Its height is given by
\[ h(t) = -4.9t^2 + 98t. \] What is the reasonable domain of this problem?

Computational Assessment Answer

Reasonable domain: [0, 20]

Conceptual Assessment

Taxonomy: B

Suppose a firework is fired straight up from the ground. Its height is given by
\[ h(t) = -4.9t^2 + 98t. \] How does changing the equation to \( h(t) = -4.9t^2 + 98t + 10 \) affect the previous situation?

Conceptual Assessment Answer

The initial height of the firework is 10 feet higher than the previous launch height; therefore, the range increases by 10.

Application (Real Life) Assessment

Taxonomy: D

Suppose a firework is fired straight up from the ground at a velocity of 98 miles/second. What is the reasonable domain of this problem?

Application (Real Life) Assessment Answer

Reasonable domain: [0, 20]
**ALIGNED COURSE EXPECTATION F07**

Solve a quadratic equation by applying the square root property.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the quadratic equation \( 2(3x + 1)^2 - 4 = 28 \) using the square root property.

**Computational Assessment Answer**

\[ x = 1, x = -\frac{5}{3} \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Solve the quadratic equation \((x + 1)^2 - 4 = c\) for \(x\) using the square root property.

**Conceptual Assessment Answer**

\[ x = -\sqrt{c+4} - 1, x = \sqrt{c+4} - 1 \]

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: C

If the flight of a ball with respect to time is modeled by the equation \( h(t) = -9.8(t + 1)^2 + 59.2 \), when will the height of the ball be 20 meters?

**Application (Real Life) Assessment Answer**

\[ t = 1 \text{ sec.} \]
**ALIGNED COURSE EXPECTATION F08**

Solve quadratic equations by factoring.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the following quadratic equation by factoring: \( 3x^2 - 5x - 2 = 0 \).

**Computational Assessment Answer**

\[ x = -\frac{1}{3}, 2 \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Determine the value(s) of \( b \) for which 2 and \( -\frac{1}{3} \) are solutions to \( 3x^2 + bx - 2 = 0 \).

**Conceptual Assessment Answer**

\( b = -5 \)

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

A rectangular garden has to have an area of 60 sq. ft. If the length is 7 feet longer than the width, find the dimensions of the garden.

**Application (Real Life) Assessment Answer**

The width is 5 ft. and the length is 12 ft.
**ALIGNED COURSE EXPECTATION F09**

Graph quadratic equations.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Graph the following quadratic function: \( y = 3x^2 - 5x - 2 \).

**Computational Assessment Answer**

![Graph of the quadratic equation](image)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Given the function \( f(x) = 3x^2 - 5x - 2 \), graph using the \( x \)-intercepts, \( y \)-intercepts, and the vertex.

**Conceptual Assessment Answer**

\( x \)-intercepts \( = (2, 0), \left(-\frac{1}{3}, 0\right) \)

\( y \)-intercept \( = (0, -2) \)

vertex \( = \left(\frac{5}{6}, -\frac{49}{12}\right) \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose the height, \( h(t) \), at a given time, \( t \), can be modeled using the quadratic function

\[
h(t) = -16t^2 + 80t + 72.
\]

Sketch a graph of this model.

Application (Real Life) Assessment Answer

![Graph of the quadratic function](image-url)
ALIGNED COURSE EXPECTATION F10

Solve quadratic equations and applications (projectile motion, geometric area, Pythagorean Theorem, etc.) using the Quadratic Formula.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following quadratic equation using the Quadratic Formula: \(3x^2 - 5x - 2 = 0\).

Computational Assessment Answer

\[ x = -\frac{1}{3}, 2 \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Using the discriminant, determine the number of real solutions of \(2x^2 - 5x + 5 = 0\).

Conceptual Assessment Answer

No real solutions.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A rectangular garden must have an area of 60 sq. ft. If the length is 7 feet longer than the width, find the dimensions of the garden using the Quadratic Formula.

Application (Real Life) Assessment Answer

The width is 5 feet, and the length is 12 feet.
● ALIGNED COURSE EXPECTATION F11

Find the $x$- and $y$-intercepts of a quadratic function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the $x$- and $y$-intercepts of $f(x) = x^2 - 2x + 1$.

**Computational Assessment Answer**

$x$-intercept (1, 0), $y$-intercept (0, 1)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Find a quadratic function with $x$-intercepts (–3, 0) and (6, 0) and a $y$-intercept of (0, –18).

**Conceptual Assessment Answer**

$f(x) = x^2 - 3x - 18$

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

John throws a baseball straight up into the air. The height, $h$, of the baseball as a function of time, $t$, in seconds is modeled by the function $h(t) = -16t^2 + 45t + 5.5$. What height was the baseball launched from? How long was the baseball in the air? Round your answer to the nearest tenth of a second.

**Application (Real Life) Assessment Answer**

Height ($y$-intercept) = 5.5; length of time in air $\approx 2.9$ seconds ($t$-intercept)
**ALIGNED COURSE EXPECTATION F12**

Solve quadratic inequalities.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the polynomial inequality $x^2 - 2x \leq 8$.

**Computational Assessment Answer**

The solution set is $\{x | -2 \leq x \leq 4\}$ or $[-2, 4]$.

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

For what portion of the domain of $f(x) = x^2 - 2x - 8$ is $f(x) \geq 0$?

**Conceptual Assessment Answer**

The portion of the domain for which $f(x) \geq 0$ is the set $\{x | -2 \leq x \leq 4\}$ or $[-2, 4]$.

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

For safety purposes, fireworks are timed to explode above a height of 40 meters. Suppose the firework is fired straight up from the ground. Its height is given by $h(t) = -4.9t^2 + 98t$.

To the nearest tenth of a second, for what range of times can the firework be safely detonated?

**Application (Real Life) Assessment Answer**

The firework may be safely detonated between 0.4 seconds and 19.6 seconds after launch.
ALIGNED COURSE EXPECTATION F13

Use the discriminant to describe the types of roots of a quadratic function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use the discriminant to determine the number and type of roots to \( f(x) = x^2 + x + 10 \).

Computational Assessment Answer

The discriminant is \(-39\). The function has two complex roots.

CONCEPTUAL ASSESSMENT

Taxonomy: C

For what values of \( b \) does the function \( f(x) = 4x^2 + bx + 36 \) have one real solution?

Conceptual Assessment Answer

This function will have one real solution when \( b = -24 \) or when \( b = 24 \).

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

An arrow is fired straight into the air from a height of 8 ft. The height, \( h \), after \( t \) seconds is given by the function \( h(t) = -16t^2 + 32t + 8 \). According to this model, will the arrow strike the ground?

Application (Real Life) Assessment Answer

Yes; examination of the discriminant shows that this function has positive, real solutions.
ALIGNED COURSE EXPECTATION F14

Find the inverse of a quadratic function using algebraic methods.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Let \( f(x) = 4x^2 + 4 \) for \( x \geq 0 \). Find \( f^{-1}(x) \).

Computational Assessment Answer

\[
f^{-1}(x) = \sqrt{\frac{x - 4}{4}}, \quad x \geq 4
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

If \( g(x) \) is the reflection of \( f(x) = \sqrt{x} \) reflected across the line \( y = x \), what is the relationship between \( f \) and \( g \)?

Conceptual Assessment Answer

\( f \) and \( g \) are the graphs of inverse functions

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A secret message is encoded using the quadratic function \( f(M) = M^2 + 4 \) for messages where \( A \to 0, B \to 1, C \to 2, \ldots, Z \to 25 \). Find the decoding function, \( g(C) \). (Hint: Find the inverse.)

Application (Real Life) Assessment Answer

\[
g(C) = \sqrt{C - 4}
\]
ALIGNED COURSE EXPECTATION F15

Verify whether a given quadratic function with a restricted domain and a square root function are inverses of each other.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Find the inverse of the function \( f(x) = 2 + \sqrt{x+5} \).

Computational Assessment Answer

\( f^{-1}(x) = x^2 - 4x - 1 \), where \( x \geq 2 \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

When finding the inverse of the function \( f(x) = 2 + \sqrt{x+5} \), why must the domain be restricted?

Conceptual Assessment Answer

The inverse of a square root function is exactly one-half of a parabola.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Prove that the standard form of a square root function \( y = \pm \sqrt{x-h} + k \) is the inverse of the standard form of a quadratic function \( y = (x-h)^2 + k \).
Application (Real Life) Assessment Answer (answers will vary)

Beginning with \( y = \pm \sqrt{x - k + h}, \)

swap \( x \) and \( y \):

\[ x = \pm \sqrt{y - k + h}, \]

and then isolate \( y \):

\[
\begin{align*}
  x-h &= \pm \sqrt{y-k} \\
  (x-h)^2 &= (\pm \sqrt{y-k})^2 \\
  (x-h)^2 &= y-k \\
  y &= (x-h)^2 + k
\end{align*}
\]
**ALIGNED COURSE EXPECTATION F16**

Graph quadratic functions.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Sketch the graph of a parabola given $x$-intercepts of $(2, 0)$ and $(4, 0)$, a $y$-intercept of $(0, -8)$, a vertex of $(3, 1)$, and an axis of symmetry.

**Computational Assessment Answer:**

![Graph of a parabola with specified characteristics](image)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Sketch the graph of the quadratic \( y = -x^2 + 6x - 8 \).

Conceptual Assessment Answer
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The height \( (h) \) of the baseball as a function of time \( (t) \) in seconds is modeled by the function \( h(t) = -16t^2 + 45t + 5.5 \). Sketch the parabola illustrating the baseball’s path using only real-world parameters.

Application (Real Life) Assessment Answer
**CONIC SECTIONS**

**ALIGNED COURSE EXPECTATION G01**

Use completing the square to transform a conic equation in general form into an equation in standard form (and vice versa).

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Use completing the square to transform the following equation into standard form; then state the center and radius:

\[ x^2 + 6x + y^2 - 16y + 57 = 0 \]

**Computational Assessment Answer**

\[ (x + 3)^2 + (y - 8)^2 = 16; \text{ center } (-3, 8), \text{ radius } 4 \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Which equation(s) below do not represent a circle?

a) \( (x-4)^2 + y^2 = -9 \)

b) \( x^2 + (y + 7)^2 = 25 \)

c) \( (x+1)^2 + (y+3)^2 = 0 \)

**Conceptual Assessment Answer**

(a) and (c)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The locations of the three receiving stations and the distances to the epicenter of an earthquake are contained in the following three equations:

\[(x - 5)^2 + (y - 4)^2 = 25, \quad (x - 3)^2 + (y - 4)^2 = 9, \quad \text{and} \quad (x + 2)^2 + (y - 4)^2 = 4.\]

Determine the location of the epicenter.

Application (Real Life) Assessment Answer

The coordinates of the epicenter are \((0, 4)\).
ALIGNED COURSE EXPECTATION G02

Graph a circle given the equation in standard or general form (no rotations).

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Graph the circle: \( x^2 + y^2 - 4x - 4y - 28 = 0. \)

Computational Assessment Answer
CONCEPTUAL ASSESSMENT

Taxonomy: C

Linda has graphed the equation \( x^2 + y^2 + 4x + 4y - 28 = 0 \) by finding the center and the radius below.

Her graph is incorrect. What mistake did she make in graphing by this method?

Conceptual Assessment Answer

Her center is at (2, 2); it should be at (–2, –2).

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
**ALIGNED COURSE EXPECTATION G03**

Graph a parabola given the equation in standard form or general form.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy:** A

Find the vertex of the parabola with the following equation in standard form:

\[ f(x) = 2(x + 1)^2 - 8. \]

Also, find the \( x \) and \( y \) intercepts of the graph and then sketch the graph.

**Computational Assessment Answer**

Vertex: \((-1, -8)\); \( x \)-intercepts: \((1, 0)\) and \((-3, 0)\); \( y \)-intercept: \((0, -6)\)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Given the graph of the parabola below, write the equation in standard form that has this graph. Also, write the equation in general form.

Conceptual Assessment Answer

\[ f(x) = (x + 3)^2 + 2 \]
\[ f(x) = x^2 + 6x + 11 \]

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
ALIGNED COURSE EXPECTATION G04

Write the equation of a circle in standard form given its attributes (center, radius, or graph).

COMPUTATIONAL ASSESSMENT

Taxonomy: A
A circle has a center at $(1, -3)$ and a radius of $5$. Write the equation of the circle in standard form.

Computational Assessment Answer

$$(x - 1)^2 + (y + 3)^2 = 25$$

CONCEPTUAL ASSESSMENT

Taxonomy: B
Explain how to write the equation of a circle given its graph.

Conceptual Assessment Answer

Answers may vary. Example: Locate the center of the circle as point $(h, k)$. Determine the radius of the circle, $r$. The equation will be in the form $(x - h)^2 + (y - k)^2 = r^2$. 
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C

Lights are placed over a stage to illuminate certain actors during a play. Technicians place each light using a coordinate plane assuming a corner of the stage is the origin. If a light must illuminate the area as seen in the graph below, what is the equation of the circle that represents where the light shines?

Application (Real Life) Assessment Answer

\((x - 5)^2 + (y - 6)^2 = 16\)
**ALIGNED COURSE EXPECTATION G05**

Write the equation of a parabola in standard form given its attributes (vertex, focus, direction, latus rectum, endpoints, etc.).

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Write an equation of a parabola with a vertex at $(1, 1)$ and directrix $x = \frac{3}{2}$.

**Computational Assessment Answer**

$$x = -\frac{1}{2} (y - 1)^2 + 1$$

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

A parabola has a directrix $y = -2$. Its focus is the point $(0, 2)$. Explain what happens to the vertex and shape of the parabola if the focus moves along the $y$-axis toward the directrix.

**Conceptual Assessment Answer**

(Wording may differ slightly.) The vertex will move down, and the parabola will narrow.

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: C

Parabolic solar panels focus the sun's energy. A cross section of a solar panel is parabolic in shape, with a pipe located at its focus. Suppose the pipe is 5 ft. from the vertex of the solar panel. Write an equation of the parabola that models the cross section of the panel (assume that the vertex is located at the origin with the $y$-axis as the axis of symmetry).

**Application (Real Life) Assessment Answer**

$$y = \frac{1}{20} x^2$$
ASSESSMENTS FOR ENTRY-LEVEL COLLEGE ALGEBRA

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FUNCTIONS AND MISCELLANY

**ALIGNED COURSE EXPECTATION A01**

Use symbols to represent unknowns and variables in problem situations.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Write the algebraic expression that the phrase represents: the sum of a number and 9.

**Computational Assessment Answer**

A letter should be used to represent “a number,” so the answer is something like

\[ x + 9 \text{ or } n + 9. \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Write a sentence to express the algebraic equation \( 5x + 3 = 9 \).

**Conceptual Assessment Answer**

The sum of 5 times a number and 3 is 9.

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

Jerica has $45 to spend for cab fare. The base charge for a cab is $5.75 and the per-mile fee is $0.30. Write an equation that could be used to find out how far she can travel with the money she has.

**Application (Real Life) Assessment Answer**

\[ 5.75 + .3x = 45 \]
**ALIGNED COURSE EXPECTATION A02**

Perform arithmetic operations on integers and rational numbers without using a calculator.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Perform the indicated operation: \(-6 + (-10)\).

**Computational Assessment Answer**

\(-6 + (-10) = -16\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Determine whether the sum of negative six and negative five is greater, less than, or equal to the product of negative eighteen and positive two-thirds.

**Conceptual Assessment Answer**

The sum of negative six and negative five is positive thirty. The product of negative eighteen and positive two-thirds is negative twelve. Positive thirty is greater than negative twelve, so the sum of negative six and negative five is greater than the product of negative eighteen and positive two-thirds.

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: B

The hottest temperature recorded in Omaha, Nebraska, was 114°F. The coldest temperature recorded was -32°F. What is the difference in the highest and lowest temperatures?

**Application (Real Life) Assessment Answer**

\(144 - (-32) = 146\), so the difference in the highest and lowest temperatures is 146°F.
ALIGNED COURSE EXPECTATION A03
Simplify algebraic expressions.

COMPUTATIONAL ASSESSMENT
Taxonomy: A
Simplify the expression by combining like terms: $6r + 5 - 4r - 3$.

Computational Assessment Answer
$6r + 5 - 4r - 3 = (6r - 4r) + (5 - 3) = 2r + 2$

CONCEPTUAL ASSESSMENT
Taxonomy: A
Write the phrase as an algebraic expression and simplify, if possible, the sum of twice a number and 4 minus the sum of a number and 5.

Conceptual Assessment Answer
$2x + 4 - (x + 5) = x - 1$

APPLICATION (REAL LIFE) ASSESSMENT
Taxonomy: B
To convert from yards to inches, we multiply the number of yards by 36. If one piece of cable is $x + 5$ yards long and a second piece is $2x - 3$ yards long, express their total length in inches as an algebraic expression. Simplify if possible.

Application (Real Life) Assessment Answer
$36(x + 5) + 36(2x - 3) = 108x + 72$
**ALIGNED COURSE EXPECTATION A04**

Use arithmetic operations to solve and manipulate equations of two variables.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy:** A

Simplify the given expression: \(4x + 6y - 5 + 14 - 3y + 12x\).

**Computational Assessment Answer**

\[
4x + 6y - 5 + 14 - 3y + 12x = 16x + 3y + 9
\]

**CONCEPTUAL ASSESSMENT**

**Taxonomy:** A

Solve the equation for \(y\): \(4x + 6y - 5 + 14 - 3y + 12x = 8\).

**Conceptual Assessment Answer**

\[
y = \frac{-1 - 16x}{3}
\]

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy:** B

The equation for converting from degrees Celsius to degrees Fahrenheit is \(F = \frac{5C}{9} + 32\).

Solve this equation so that it can be used to convert from degrees Fahrenheit to degrees Celsius.

**Application (Real Life) Assessment Answer**

\[
C = \frac{9(F - 32)}{5}
\]
ALIGNED COURSE EXPECTATION A05

Transform between representations of functions (e.g., table of values, equation, graph, verbal description).

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use the given equation to complete the table of values: $9x + 3y = 18$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>0</td>
</tr>
</tbody>
</table>

Computational Assessment Answer

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
CONCEPTUAL ASSESSMENT

Taxonomy: C

Use the given graph to complete the table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>2</td>
</tr>
</tbody>
</table>

Conceptual Assessment Answer

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>
**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: B**

The table below gives the career batting averages of some famous professional baseball players. Write this data as a set of ordered pairs.

<table>
<thead>
<tr>
<th>Player</th>
<th>Career Batting Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ty Cobb</td>
<td>.366</td>
</tr>
<tr>
<td>Babe Ruth</td>
<td>.342</td>
</tr>
<tr>
<td>Lou Gehrig</td>
<td>.340</td>
</tr>
<tr>
<td>Joe DiMaggio</td>
<td>.325</td>
</tr>
<tr>
<td>Ted Williams</td>
<td>.344</td>
</tr>
</tbody>
</table>

**Application (Real Life) Assessment Answer**

```latex
\{(Ty \, Cobb, \, 0.366), (Babe \, Ruth, \, 0.342), (Lou \, Gehrig, \, 0.340), (Joe \, DiMaggio, \, 0.325), (Ted \, Williams, \, 0.344)\}
```
**ALIGNED COURSE EXPECTATION A06**
Evaluate functions given an input value.

**COMPUTATIONAL ASSESSMENT**
Taxonomy: A
Evaluate $g(x) = 2x^3 - 5x^2 + 3x - 1$ for $x = 5$.

Computational Assessment Answer

$f(5) = 139$

**CONCEPTUAL ASSESSMENT**
Taxonomy: B
Evaluate $g(x) = 2x^2 - 4x$ for $x = a + 2$.

Conceptual Assessment Answer

$g(a + 2) = 2a^2 + 4a$

**APPLICATION (REAL LIFE) ASSESSMENT**
Taxonomy: C
The revenue, in dollars, for selling $x$ basketball goals is given by $R(x) = 25x$. The cost, in dollars, for producing $x$ basketball goals is given by $C(x) = 5x + 105$. The profit, in dollars, from selling $x$ basketball goals is computed by subtracting the cost of producing $x$ basketball goals from the revenue generated by selling $x$ basketball goals. Find the profit for selling 15 basketball goals.

Application (Real Life) Assessment Answer

$195$
ALIGNED COURSE EXPECTATION A07

Transform between zeros and x-intercepts of functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

The function \( f(x) \) has zeros of \{-2, 4, 5\}. State the x-intercepts for the graph of \( f(x) \).

Computational Assessment Answer

x-intercepts: (-2, 0), (4, 0), and (5, 0)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Let \( f(x) \) be a function and \( a \) be a real number such that \( f(a) = 0 \). Give one x-intercept of \( f(x) \).

Conceptual Assessment Answer

x-intercept: \( (a, 0) \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Consider the graph below, which shows the relationship between the profit, \( P \), on the vertical axis and the units sold, \( x \), on the horizontal axis.

How many values for \( x \) result in a zero profit?

Application (Real Life) Assessment Answer

2
ALIGNED COURSE EXPECTATION A08

Given a table, mapping diagram, or graph, determine the domain and range.

COMPUTATIONAL ASSESSMENT:

Taxonomy: A

Give the domain and range of the relation represented in the graph below using interval notation.

Computational Assessment Answer

Domain: [-4, 3]

Range: [-4, 4]

CONCEPTUAL ASSESSMENT

No assessment.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

Consider the following plot of home prices (vertical axis) against home size (horizontal axis). Represent the domain and range of this relation as sets.

Application (Real Life) Assessment Answer

Domain: \{850; 900; 1,200; 1,400; 2,500; 5,000\}

Range: \{$78,000; $120,000; $135,000; $240,000; $285,000; $330,000; $650,000\$\}
ALIGNED COURSE EXPECTATION A09

Describe functional relationships for given problem situations, and write equations or inequalities that could be used to solve real world problems.

COMPUTATIONAL AND CONCEPTUAL ASSESSMENTS

No assessment.

APPLICATION (REAL LIFE) ASSESSMENT 1

Taxonomy: C

The cost to manufacture bottles of water is $0.50 per bottle. The cost to ship each bottle is $0.25. Write a function, \( C \), for the cost of manufacturing and shipping \( x \) bottles.

Application (Real Life) Assessment Answer 1

\[ C(x) = 0.75x \]

APPLICATION (REAL LIFE) ASSESSMENT 2

Taxonomy: C

The cost to manufacture bottles of water is $0.50 per bottle. The cost to ship each bottle is $0.25. Set up an equation that could be solved to determine the number of bottles that could be manufactured and shipped for $21,000.

Application (Real Life) Assessment Answer 2

\[ 0.75x = 21000 \]
APPLICATION (REAL LIFE) ASSESSMENT 3

Taxonomy: C

The cost to manufacture bottles of water is $0.50 per bottle. The cost to ship each bottle is $0.25. If a company has only $24,000 available to produce and ship water bottles, write an inequality that could be used to determine the largest number of bottles than can be produced.

Application (Real Life) Assessment Answer 3

\[0.75x \leq 24000\]
ALIGNED COURSE EXPECTATION A10

Discriminate between constant, linear, and quadratic parent functions and their graphs.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

State the parent function for each of the following functions, and write the parent function in function notation.

a) \( f(x) = (x-2)^2 + 14 \)

b) \( g(x) = -4x + 1 \)

c) \( h(x) = -14 \)

Computational Assessment Answer

a) Quadratic

b) Linear

c) Constant

CONCEPTUAL ASSESSMENT

Taxonomy: C

Do any of the following functions have the same parent function? Explain.

a) \( f(x) = 2x^2 + 4 \)

b) \( f(x) = 2(x+1)^2 + 4 \)

c) \( f(x) = 2x + 4 \)
Conceptual Answer Assessments

Equations (a) and (b) have the same parent function: they are both quadratic functions. Equation (c) has a linear parent function.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

The following scatter plot shows the relationship between revenue earned (on vertical axis) and various levels of production (on the horizontal axis). Which parent function—quadratic, linear, or constant—best models this situation?

Application (Real Life) Assessment Answer

Quadratic
SYSTEMS

ALIGNED COURSE EXPECTATION B01

Convert verbal situations to linear systems of equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Mary, Joan, and Katy spent a total of $49 renting movies last month. Mary and Katy’s expenditure totaled three-fourths of Joan’s. If Katy spent $10 more than Mary, how much did each spend?

Create a system of equations to solve this problem. Use $M$, $J$, and $K$ to represent the amounts that Mary, Joan, and Katy, respectively, spent.

Computational Assessment Answer

\[
\begin{align*}
&M + J + K = 49 \\
&M + K = \frac{3}{4}J \\
&K = M + 10
\end{align*}
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Mark, Joey, and Tim ate at restaurants a total of 54 times last month. Joey and Tim’s times totaled one-half of Mark’s. If Tim ate out two fewer times than Joey, how many times did each eat out?

Using $M$, $J$, and $T$ to represent the number of times Mark, Joey, and Tim, respectively, ate out, a student gave the following system of equations to solve the problem:

\[
\begin{align*}
&M + J + T = 54 \\
&M + T = \frac{1}{2}J \\
&T = 2 - J
\end{align*}
\]

There is an error in the system. What mistake was made?
Conceptual Assessment Answer

The third equation should be \( T = J - 2 \).

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A cashier at Astroworld amusement park has a total of $2,480, made up of five, ten, and twenty dollar bills. The total number of bills is 290, and the value of the tens is $60 more than the value of the twenties. How many of each type of bill does the cashier have?

Application (Real Life) Assessment Answer

The cashier has 164 five dollar bills, 86 ten dollar bills, and 40 twenty dollar bills.
EXponential & Logarithmic Expressions & Equations

**ALIGNED COURSE EXPECTATION C01**

Simplify exponential expressions by applying laws of exponents.

**Computational Assessment**

**Taxonomy: A**

Simplify each exponential expression. Leave your answer with only positive exponents.

a) \(5^3 \times 5^4\)

b) \(\frac{a^3 b^3 c^3}{a^2 b^3 c^4}\)

c) \(\left(\frac{2a^{-2}b}{c^{-3}}\right)^{-2}\)

**Computational Assessment Answer**

a) \(5^7\)

b) \(\frac{a}{c^2}\)

c) \(\frac{a^4 c^{-6}}{4b^2}\)
CONCEPTUAL ASSESSMENT

Taxonomy: C

a) Is the exponential operation commutative? For any real numbers $a$ and $b$, does

$$a^b = b^a?$$

b) Are the rules of exponents associative? For any real numbers $a$, $b$, and $c$, does

$$\left(a^b\right)^c = a^{bc}?$$

Conceptual Assessment Answer

a) No. In general $a^b \neq b^a$.

b) Yes. $\left(a^b\right)^c = a^{bc}$.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

This speed of sound in air is approximately $(1.125) 10^3$ feet per second. Use scientific notation to express the distance that sound would travel in 9 seconds.

Application (Real Life) Assessment Answer

$(1.125) 10^4$ feet
RADICAL EXPRESSIONS AND EQUATIONS

• ALIGNED COURSE EXPECTATION E01
Simplify radical expressions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Simplify the expression \(\frac{-6 + \sqrt{-24}}{12}\) and write in standard form.

Computational Assessment Answer

\[ \frac{1}{2} + \frac{\sqrt{6}}{6} \]

CONCEPTUAL ASSESSMENT

Taxonomy: B

Let \(x\) be a positive real number. Simplify \(\sqrt{16x^2}\).

Conceptual Assessment Answer

\(4x\)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Find a value of $c$ for which $\sqrt{6+c}x$ will simplify to $3\sqrt{x}$.

Application (Real Life) Assessment Answer

$c = 3$
ALIGNED COURSE EXPECTATION E02

Solve square root equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following radical equation for $x$: $\sqrt{x+5}-4=-2$.

Computational Assessment Answer

$x = -1$

CONCEPTUAL ASSESSMENT

Taxonomy: B

How many real solutions are there to $\sqrt{x}=-2$?

Conceptual Assessment Answer

This equation has no solution. The left-hand side is a positive quantity while the right-hand side is a negative quantity.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The function $v(d) = \sqrt{19.62d}$ gives the instantaneous velocity in feet per second of a freely falling object that has traveled distance $d$ feet. How far must a freely falling object fall to reach a velocity of 12 feet/second? (Round your answer to the nearest tenth of a foot.)

Application (Real Life) Assessment Answer

The object must fall 7.3 feet.
POLYNOMIAL EXPRESSIONS & EQUATIONS

ALIGNED COURSE EXPECTATION F01
Write polynomial expressions in standard format, \( ax^n + bx^{n-1} + cx^{n-2} + \ldots \).

COMPUTATIONAL ASSESSMENT
Taxonomy: A
Write the following polynomial expression in standard form: \( 4x^3 + x^5 - 7 + 3x^2 \).

Computational Assessment Answer
\( x^5 + 4x^3 + 3x^2 - 7 \)

CONCEPTUAL ASSESSMENT
Taxonomy: C
Expand the following polynomial expression and express your answer as a single polynomial in standard form: \( (2x + 1)^3 \).

Conceptual Assessment Answer
\( 8x^3 + 12x^2 + 6x + 1 \)

APPLICATION (REAL LIFE) ASSESSMENT
No assessment.
ALIGNED COURSE EXPECTATION F02
Find the greatest common factor of a polynomial expression.

COMPUTATIONAL ASSESSMENT
Taxonomy: A
What is the greatest common factor of the following polynomial?

\[4x^2 + 12x + 16\]

Computational Assessment Answer
4

CONCEPTUAL ASSESSMENT
Taxonomy: C
What could the value of \( b \) be if 4 is the greatest common factor of the following polynomial?

\[4x^2 + bx + 16\]

Conceptual Assessment Answer
Any multiple of 4.

APPLICATION (REAL LIFE) ASSESSMENT
No assessments.
**ALIGNED COURSE EXPECTATION F03**

Graph linear equations.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy:** A

Graph a line given a slope of \( \frac{1}{3} \) and a y-intercept of \(-2\).

**Computational Assessment Answer**

![Graph of a line with a slope of \( \frac{1}{3} \) and a y-intercept of \(-2\).]
CONCEPTUAL ASSESSMENT

Taxonomy: A

Graph the equation $2x - 6y = 12$.

Conceptual Assessment Answer
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

What is the relationship between the equations \( y = 2x + 6 \) and \( y = \frac{1}{2}x - 4 \)? Graph these equations.

Application (Real Life) Assessment Answer

The two equations are perpendicular lines.
ALIGNED COURSE EXPECTATION F04

Write a linear equation given the graph of a line.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

What are the slope and $y$-intercept of this graph?

Computational Assessment Answer

Slope: $-\frac{1}{2}$

$y$-intercept is (0, 1)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Write the equation of this line:

Conceptual Assessment Answer

\[ y = -\frac{1}{2}x + 1 \]

APPLICATION (REAL LIFE) ASSESSMENT

No assessments.
**ALIGNED COURSE EXPECTATION F05**

Determine the $x$- and $y$-intercepts of a linear function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Determine the $x$- and $y$-intercepts of the following graph of a linear function.

**Graph Image**

- **Computational Assessment Answer**
  
  $x$-intercept: (2, 0)
  
  $y$-intercept: (0, 1)


CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose the graph of a linear function has a positive $y$-intercept and a positive $x$-intercept. What can be said about the slope of this linear function?

Conceptual Assessment Answer

It must be negative.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The straight-line depreciation of a truck can be modeled by the linear equation $v(x) = -2000x + 30000$, where $v$ is the value of the truck after $x$ years. What does the $x$-intercept of this graph represent?

Application (Real Life) Assessment Answer

The number of years at which the truck will have no value remaining.
ALIGNED COURSE EXPECTATION F06

Given the graph of a quadratic equation, students can find the $y$-intercept.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

What is the $y$-intercept of the graph?

Computational Assessment Answer

$(0, -7)$
CONCEPTUAL ASSESSMENT

Taxonomy: C

Does every quadratic function have a y-intercept? How do you know?

Conceptual Assessment Answer

Yes, because the domain of any quadratic is \((-\infty, \infty)\), so the graph will cross the y-axis.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The height in feet, \(h(t)\), of a projectile after \(t\) seconds can be modeled by \(h(t) = -16t^2 + 32t + 5\). The y-intercept of this function is \((0, 5)\). What does this represent in the model?

Application (Real Life) Assessment Answer

The initial height of the projectile \((at t = 0)\).
ALIGNED COURSE EXPECTATION F07

Given the graph of a quadratic equation, students can find the $x$-intercept(s).

COMPUTATIONAL ASSESSMENT

Taxonomy: A

What are the $x$-intercepts of the graph below?

Computational Assessment Answer

The $x$-intercepts are $(-2, 0)$ and $(3, 0)$.
CONCEPTUAL ASSESSMENT

Taxonomy: C
Does every quadratic function have an $x$-intercept? Explain.

Conceptual Assessment Answer
No, since a parabola can open upward above the $x$-axis. Such a parabola has no $x$-intercept.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D
The height in feet, $h(t)$, of a projectile after $t$ seconds can be modeled by
$h(t) = -16t^2 + 32t$. The $t$-intercepts of this function are $(0, 0)$ and $(2, 0)$. What do these represent in the model?

Application (Real Life) Assessment Answer
The $t$-intercepts represent the times at which the projectile is on the ground.
ALIGNED COURSE EXPECTATION F08

Given a graph of a quadratic function, find the domain and range.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

What are the domain and range of the graph below?

Computational Assessment Answer

Domain: \((-\infty, \infty)\)

Range: \([-3, \infty)\)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose the domain of a quadratic function contains only positive numbers. What can be said about the graph of this quadratic function?

Conceptual Assessment Answer

The vertex lies in the upper half of the plane and the parabola opens upward.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The height in feet, \( h(t) \), of a projectile after \( t \) seconds can be modeled by \( h(t) = -16t^2 + 64 \). Consider the graph below. What is the reasonable domain and range of this model?

Application (Real Life) Assessment Answer

Reasonable domain: \([0, 2]\)

Reasonable range: \([0, 64]\)
ALIGNED COURSE EXPECTATION F09

Solve quadratic equations by factoring.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following quadratic equation by factoring \( x^2 - x - 6 = 0 \).

Computational Assessment Answer

\[ x = 3, -2 \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Determine the value(s) of \( b \) for which 2 and \(-3\) are solutions to \( x^2 + bx - 6 = 0 \).

Conceptual Assessment Answer

\[ b = -1 \]

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A rectangular garden has to have an area of 60 square feet. If the length is 7 feet longer than the width, find the dimensions of the garden.

Application (Real Life) Assessment Answer

Width: \( w = 5 \)
Length: \( l = 12 \)
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ASSESSMENTS FOR
EXIT-LEVEL
COLLEGE ALGEBRA

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FUNCTIONS & MISCELLANY

**ALIGNED COURSE EXPECTATION A01**

Identify the parent function (square, cube, square root, cube root, absolute value, reciprocal, exponential/logarithmic) and transformations (translations, reflections, dilations) for a given transformed function and use them to graph the functions.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Consider the following equation:

\[ y = -(x - 3)^3 + 1 \]

a) This is a shift of which power function?

b) What is the vertical shift?

c) What is the horizontal shift?

d) What else has been done to the parent function?

e) Sketch a graph of the function, including any x-intercepts.

**Computational Assessment Answer**

a) \( y = x^2 \)

b) Up 1

c) Right 3

d) Horizontal reflection

e) (see graph)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Consider the following function: \( y = \ln x^2 \)

This function has appeared on your homework with a request to graph it. Your friend says, “Oh, I know how to do this!” She then writes the following:

\[ y = \ln x^2 = 2 \ln x \]

Then she says, “Now it’s just a stretch of the function \( y = \ln x \) by a factor of 2!”

The principle your friend is using is a rule you find in your notes: \( \log_a b^c = c \log_a b \).

a) So, is your friend right? Are the graphs of the two functions the same?

b) What is missing from the rule in your notes?

**Conceptual Assessment Answer**

a) No, they are not the same. The domains are different.

b) The domain restriction; the rule holds for \( b > 0 \).
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

A small town in Kansas is experiencing population decline that seems to follow the exponential decay rule:

\[ P(t) = P_0 e^{kt}. \]

The population in 1910 was 1,600 people. In 1960, the population had fallen to 975 people. If we assume the population is following the above model:

a) What is the decay constant \( k \)?

b) What is the half-life?

c) What will the population be in 2020?

d) Graph the population levels from 1910 to 2020.

Application (Real Life) Assessment Answer

a) \(-0.0099\)

b) 70 years

c) 656 people

d) Graph showing population levels from 1910 to 2020.
**ALIGNED COURSE EXPECTATION A02**

Evaluate a given piecewise defined function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: C

Evaluate the function at $x = 3$:

$$f(x) = \begin{cases} 
-x + 2 & \text{if } 0 \leq x \leq 2 \\
\frac{1}{2}x + 1 & \text{if } 2 < x < 6 
\end{cases}$$

Computational Assessment Answer

$$f(3) = \frac{1}{2}(3) + 1 = \frac{5}{2}$$

**CONCEPTUAL ASSESSMENT**

No assessment.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B
Your income tax rate is 0% on the first $10,000 earned, then 10% on the next $90,000, then 15% on all money earned above $100,000. This is modeled in the piecewise function below.

\[ T(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 10,000 \\
.1x - 1000 & \text{if } 10,000 \leq x \leq 100,000 \\
.15x - 6000 & \text{if } 100,000 \leq x
\end{cases} \]

Use \( T(x) \) to find your income tax if you made $28,000.

Application (Real Life) Assessment Answer

\[ T(28000) = .1(28000) - 1000 = 1800 \]

The tax on $28,000 income is $1,800.
**ALIGNED COURSE EXPECTATION A03**

Determine the intervals on which a given function is increasing, decreasing, and constant from a graph.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: C**

Consider the following graph of a function $f(x)$:

The scale on both the $x$- and $y$-axis is 1 square = 1 unit.

What is $f(2) - f(7)$?

**Computational Assessment Answer**

-4
CONCEPTUAL ASSESSMENT:

Taxonomy: C

Consider the graph of a function \( g(x) \):

The scale on the \( x \)-axis is 1 square = 1 unit, but you do not know the scale on the \( y \)-axis. You do, however, know that \( g(-7) - g(7) = 60 \). What is \( g(-1) \)?

Conceptual Assessment Answer

\( g(-1) = 80 \)
APPLICATION (REAL LIFE) ASSESSMENT

Consider the following graph for a profit function:

The \( x \)-axis is in hundreds of units produced per day. The \( y \)-axis is in thousands of dollars of profit per day.

a) How many units should be produced per day to maximize the profit?

b) What is the maximum profit that can be produced?

c) How much profit will you lose if you run the facility at its maximum rate of 800 units per day?

Application (Real Life) Assessment Answer

a) 500 units per day

b) $4,000 per day

c) $2,000 per day
**ALIGNED COURSE EXPECTATION A04**

Determine whether a given function is even, odd, or neither.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: C**

Identify the functions as even, odd, or neither:

a) \( f(x) = x^2 - 4 \)
b) \( f(x) = x^3 - 4 \)
c) \( f(x) = x^3 - 4x \)
d) \( f(x) = \frac{x^3 - 4}{x^3} \)

**Computational Assessment Answer**

a) Even
b) Neither
c) Odd
d) Odd

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Suppose \( f \) is an even function and \( g \) is an odd function. Identify the following as even, odd, or neither:

a) \( 2f(x) \)
b) \( 3f(x) \)
c) \( f(x) + g(x) \)
d) \( f(x) \times g(x) \)
Conceptual Assessment Answer

a) Even
b) Even
c) Neither
d) Odd

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C

Below is a graph of the function $f(x) = \sin(x)$, a function used in trigonometry:

Based on the graph, do you think the function is even, odd, or neither?

Application (Real Life) Assessment Answer

Odd
**ALIGNED COURSE EXPECTATION A05**

Find the composition of two functions.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: C

For the two functions \( f(x) = x^2 - x \) and \( g(x) = 2x - 1 \), find \( f \circ g(x) \).

Computational Assessment Answer

\[ f \circ g(x) = x \]

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

For the two functions \( f(x) = \ln(x - 3) \) and \( g(x) = e^x + 3 \), find \( f \circ g(x) \).

Conceptual Assessment Answer

\[ f \circ g(x) = x \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

When you toss a pebble into a pond, you observe that the radius of one of the rings is growing at a rate of 3 centimeters per second, so the radius as a function of time is \( r(t) = 3t \). You know that the area of a circle can be described by the equation \( A(r) = \pi r^2 \).

Use composition of functions to find the area as a function of time.

Application (Real Life) Assessment Answer

\[ A(t) = 9\pi t^2 \]
ALIGNED COURSE EXPECTATION A06

Decompose a given function into a composition of simpler functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: C

Decompose the function \( h(x) = (x^2 + 2x + 3)^3 \) into two functions \( f(x) \) and \( g(x) \), such that \( h(x) = f(x) \circ g(x) \).

Computational Assessment Answer

\[
\begin{align*}
  f(x) &= x^3 \\
  g(x) &= x^2 + 2x + 3 
\end{align*}
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

For the function \( h(x) = \sqrt{x^2 + 3} \), one decomposition of this is \( f(x) = x^4 \) and \( g(x) = x^2 + 3 \). Find a different pair of functions \( f \) and \( g \) that could form this composition.

Conceptual Assessment Answer

\[
\begin{align*}
  f(x) &= \sqrt{x + 3} \\
  g(x) &= x^2 
\end{align*}
\]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The ripple when a rock is thrown into a pond moves at 3 feet per second, so the radius of the circle formed by the ripple is given by $r(t) = 3t$. Find the composition function that represents the area of the circular ripple at time $t$.

Application (Real Life) Assessment Answer

$A(t) = \pi (3t)^2$
**ALIGNED COURSE EXPECTATION A07**

Given a relation, determine the domain and range.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Give the domain and range of the relation represented by the equation \((x - 3)^2 + (y + 2)^2 = 16\).

**Computational Assessment Answer**

Domain: \([-1, 7]\)

Range: \([-6, 2]\)

**CONCEPTUAL ASSESSMENT**

No assessment.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

Consider the following plot of home prices (vertical axis) against home size (horizontal axis). Represent the domain and range of this relation as sets.

Application (Real Life) Assessment Answer

Domain: \{850; 900; 1,200; 1,400; 2,500; 5,000\}

Range: \{78,000; 120,000; 135,000; 240,000; 285,000; 330,000; 650,000\}
ALIGNED COURSE EXPECTATION A08

Simplify algebraic expressions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Simplify the following expression by combining like terms:

\[5x^3 - 12y^2 + 8x^2y - 2x^3 + 7y^2x + 9x - 4x^2y - 7y^2 - 3x + 15\]

Computational Assessment Answer

\[3x^3 - 19y^2 + 4x^2y + 7y^2x + 6x + 15\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Simplify the expression below by combining like terms:

\[2(x^2 - 4x^2 + 8x - 9) - 3(3x^2 + 5x)\]

Conceptual Assessment Answer

\[2x^3 - 17x^2 + x - 18\]

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

The number of tickets sold to a baseball game on Thursday is expressed by \(2x\). The number of tickets sold on Friday is expressed by \(5x + 2\). The number of tickets sold on Saturday is expressed by \(8x - 4\). Express the total number of tickets sold as an algebraic expression.

Application (Real Life) Assessment Answer

\[15x - 2\]
ALIGNED COURSE EXPECTATION A09

Determine symmetry of a relation given the graph or equation.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Determine whether each of the functions given below is:

a) symmetric about the y-axis

b) symmetric about the origin

c) symmetric about a vertical axis of symmetry

d) none of the above

1. \( f(x) = 2x^2 - 5x + 11 \)

2. \( g(x) = x^5 - 12 \)

3. \( f(x) = x\sqrt{x^4} + 5 \)

4. \( j(x) = \frac{2\sqrt{x}}{x} \)

Computational Assessment Answer

1. c

2. d

3. b

4. a and c
CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose \( f \) is an even function—that is, \( f(-x) = f(x) \) for all \( x \) in the domain of \( f \). If \( (-2, 17) \) is a point in the graph of \( f \), give another point in the graph.

Conceptual Assessment Answer

\((2, 17)\) is another point in the graph

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Adam is a pyrotechnician working on a fireworks display for an upcoming concert. He has a new mortar launcher that fires mortars along a parabolic trajectory to a height of 140 feet. For the fireworks he plans to use, the optimal detonation height is above 110 feet. If it takes the fireworks 2 seconds to travel from 110 feet to 140 feet, how many seconds does Adam have to detonate his fireworks at or above 110 feet?

Application (Real Life) Assessment Answer

4 seconds
ALIGNED COURSE EXPECTATION A10

Graph a given piecewise defined function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Graph the piecewise function given by \( f(x) = \begin{cases} 
2x + 1 & \text{if } x < 1 \\
-x^2 + 4 & \text{if } x \geq 1
\end{cases} \).

Computational Assessment Answer

CONCEPTUAL ASSESSMENT

No assessment.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Cynthia’s Bakery sells cupcakes for $3 each. If you buy two dozen or more, the cost of each cupcake is discounted by 50¢. Graph a piecewise function that gives the total cost, \( C \), of purchasing \( x \) cupcakes.

Application (Real Life) Assessment Answer
ALIGNED COURSE EXPECTATION A11
Perform basic operations involving complex numbers.

COMPUTATIONAL ASSESSMENT
Taxonomy: A
Simplify the following expressions to a single complex number of the form:

a) \((2+5i) + (-8+3i)\)

b) \((4-5i) - (2-13i)\)

c) \((4+2i)(3-7i)\)

d) \(\frac{2+3i}{5-4i}\)

Computational Assessment Answer
a) \(-6+8i\)

b) \(2+8i\)

c) \(26-22i\)

d) \(\frac{2}{41} + \frac{23}{41}i\)
CONCEPTUAL ASSESSMENT

Taxonomy: C

The complex number \(2 - i\) is a solution of the equation \(x^2 - 4x + 5 = 0\).

a) Write down a second solution of the equation.

b) Verify/show that your answer of part (a) is a solution of the equation.

Conceptual Assessment Answer

a) Complex solutions of polynomial equations occur in complex conjugate pairs. Therefore, if \(2 - i\) is a solution, then a second solution must be \(2 + i\).

b) Substitution of \(-1 + i\) into the equation (below) verifies it is a solution.

\[
(2 + i)^2 - 4(2 + i) + 5 = (2 + i)(2 + i) - 4(2 + i) + 5 \\
= 4 + 2i + 2i + i^2 - 8(2 + i) + 5 \\
= -1 + 1 \\
= 0
\]

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
**SYSTEMS**

**ALIGNED COURSE EXPECTATION B01**

Solve a system of linear equation using Gauss-Jordan Elimination.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the following system of equations using Gauss-Jordan Elimination:

\[
\begin{align*}
    x - 2y + 3z &= -4 \\
    3x + y - z &= 0 \\
    2x + 3y - 5z &= 1
\end{align*}
\]

**Computational Assessment Answer**

\((-1, 6, 3)\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

As part of her solution using Gauss-Jordan Elimination, Mary has obtained the following matrix:

\[
\begin{bmatrix}
    1 & 1 & 2 & | & 3 \\
    0 & 2 & 3 & | & 4 \\
    0 & 0 & 2 & | & 5
\end{bmatrix}
\]

To get a 1 in the pivot position for column two, Mary decides to interchange rows one and two. Explain why this is not an appropriate step.

**Conceptual Assessment Answer**

It will change column one.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Fred and Mary buy snacks at the coffee bar. On the first day they buy one cup of coffee, one kolache, and two donuts for $7.25. On the second day they buy two coffees and three donuts for $11.25. On the third day they buy two coffees, one kolache, and one donut. What are the prices for one coffee, one kolache, and one donut?

Application (Real Life) Assessment Answer

The price of one coffee is $4.50, the price of one kolache is $1.25, and the price for one donut is 75¢.
ALIGNED COURSE EXPECTATION B02

Distinguish between inconsistent, dependent, and independent systems of linear equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use Gauss-Jordan Elimination on the following system of equations to find the row-reduced form of the system's augmented matrix and determine how many (if any) solutions exist.

Computational Assessment Answer

\[
\begin{bmatrix}
1 & 0 & 2 & | & 3 \\
0 & 1 & 3 & | & 1 \\
0 & 0 & 0 & | & 0
\end{bmatrix}; \text{ infinite number of solutions}
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C

As part of his solution to three problems, Robert has reduced his three augmented matrices to the following:

a) \[
\begin{bmatrix}
1 & 0 & | & 3 \\
0 & 1 & | & 4
\end{bmatrix}
\]

b) \[
\begin{bmatrix}
1 & 6 & | & 2 \\
0 & 0 & | & 1
\end{bmatrix}
\]

c) \[
\begin{bmatrix}
1 & 6 & | & 2 \\
0 & 0 & | & 3
\end{bmatrix}
\]

Which of the above reduced forms correspond to a unique solution, no solution, and infinite number of solutions?
Conceptual Assessment Answer

a) corresponds to a unique solution

b) corresponds to an infinite number of solutions

c) corresponds to no solution

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Use an augmented matrix with Gauss-Jordan Elimination to solve the following:

Farmer Joe wishes to keep sheep, goats, and pigs. Each sheep needs 1 bale of hay, 3 bushels of cracked corn, and 2 ounces of wormer. Each pig needs 0 bales of hay, 1 bushel of cracked corn, and 2 ounces of wormer. Each goat needs 2 bales of hay, 2 bushels of cracked corn, and 6 ounces of wormer. Farmer Joe has 5 bales of hay, 18 bushels of cracked corn, and 18 ounces of wormer. How many sheep, pigs, and goats can Farmer Joe have if he wishes to meet all needs and completely use all his feed and wormer?

Application (Real Life) Assessment Answer

\[
\begin{bmatrix}
1 & 0 & 2 & | & 5 \\
0 & 1 & 1 & | & 3 \\
0 & 0 & 0 & | & 2 \\
\end{bmatrix}
\]

no solution
ALIGNED COURSE EXPECTATION B03

Describe the (infinitely many) solutions to dependent systems of linear equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following system of equations using Gauss-Jordan Elimination:

\[
\begin{align*}
2x - 4y - 3z &= 3 \\
x + 3y + z &= -1 \\
4x - 2y - z &= 1
\end{align*}
\]

Computational Assessment Answer

\( (0.5 + 0.5z, -0.5 - 0.5z, z) \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Given the following system of equations:

\[
\begin{align*}
x + 3y + z &= -1 \\
2x - 4y - 3z &= 3'
\end{align*}
\]

explain why this system has an infinite number of solutions without solving the system.

Conceptual Assessment Answer

Every linear system has either one solution, no solution, or an infinite number of solutions. Since the system has three variables but only two equations, it has either no solution or an infinite number of solutions. The second equation is not a constant multiple of the first, and therefore, there are an infinite number of solutions.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The figure below shows the intersections of three one-way streets. This situation is modeled by the linear system

\[
\begin{align*}
x + 8 &= y + 14 \\
z + 11 &= x + 9 \\
y + 23 &= z + 19
\end{align*}
\]

where \(x\) is the number of cars per minute passing between intersections one and two, \(y\) is the number of cars per minute passing between intersections one and three, and \(z\) is the number of cars passing between intersections two and three. Find an expression for the (infinitely many) solutions to this system.

Application (Real Life) Assessment Answer

The solution set is given by \((z + 2, z - 4, z)\).
EXponential & Logarithmic Expressions & Equations

● Aligned Course Expectation C01

Find inverses of logarithmic and exponential functions.

Computational Assessment

Taxonomy: A

Find each inverse:

a) \( f(x) = \frac{5^{4x+1}}{2} \)

b) \( f(x) = \frac{3^{6x+5}}{3^{8x+6}} - 4 \)

c) \( f(x) = \log_{\frac{1}{3}} x + 4 \)

d) \( f(x) = 6\log_{3} (-4x) + 7 \)

Computational Assessment Answer

a) \( f^{-1}(x) = \frac{\log_{3} (2x) - 1}{4} \)

b) \( f^{-1}(x) = \frac{\log_{3} (x + 4) + 1}{2} \)

c) \( f^{-1}(x) = \frac{1}{5^{x-4}} \)

d) \( f^{-1}(x) = \frac{1}{5^{x-4}} \)
CONCEPTUAL ASSESSMENT

Taxonomy: C

If an exponential function gives the population at a given time, then what does the inverse of the exponential function tell you?

Conceptual Assessment Answer

It gives you the time when the population is a certain value.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Estimates of the amounts (in billions of dollars) of U.S. online advertising spending from 2007 to 2011 is modeled by the exponential function \( S = 10.33 e^{0.1022t} \) for \( 7 \leq t \leq 11 \), where \( S \) is the amount of spending (in billions) and \( t = 7 \) represents 2007. Find the inverse of this exponential model.

Application (Real Life) Assessment Answer

\[
S^{-1}(t) = \frac{\ln(t) - 10.33}{0.1022} = 9.7847 \ln\left(\frac{t}{10.33}\right), \quad \text{or} \quad t(S) = 9.7847 \ln\left(\frac{S}{10.33}\right), \quad 21.12 \leq S \leq 31.79
\]
**ALIGNED COURSE EXPECTATION C02**

Find domain and range of exponential and logarithmic functions.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Give the domain and range for each of the following exponential functions:

a) \( f(x) = e^{x-2} \)

b) \( f(x) = e^x - 2 \)

c) \( f(x) = 2 - e^{x-2} \)

**Computational Assessment Answer**

a) Domain: \((-\infty, \infty)\), or \(x\) can be any real number; range: \((0, \infty)\) or \(y > 0\)

b) Domain: \((-\infty, \infty)\), or \(x\) can be any real number; range: \((-2, \infty)\) or \(y > -2\)

c) Domain: \((-\infty, \infty)\), or \(x\) can be any real number; range: \((-\infty, 2)\) or \(y < 2\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

The bacteria in a petri dish double every three minutes. If the petri dish is full of bacteria after 40 minutes, how long did it take before the petri dish was one-quarter full?

**Conceptual Assessment Answer**

The petri dish was half-full after 37 minutes and one-quarter full after 33 minutes.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Let $A$ represent the mass of Fermium−253 (253Fe) (in grams), whose half−life is three days.

The quantity of Fermium−253 present after $t$ days is $A = 4 \times \left(\frac{1}{2}\right)^{t/3}$.

a) Determine the initial quantity (when $t = 0$).

b) Determine the quantity present after 9 days.

c) Determine the range of the function that corresponds to the domain $t = 0$ to $t = 12$.

Application (Real Life) Assessment Answer

a) 4 grams

b) 0.5 grams

c) [0.25, 4]
ALIGNED COURSE EXPECTATION C03

Solve exponential equations by applying one-to-one property.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following exponential equation:

\(-2(4^{-2x}) + 13 = 5\).

Computational Assessment Answer

\[ x = -\frac{1}{2} \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Determine the value for \(c\) so that the equation has a solution of \(x = 5\):

\[ c^{2x-2} = 9^{9-x} \]

Conceptual Assessment Answer

\(c = 3\) or \(c = -3\)

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose that the population of a city is 20,000 at the beginning of 2000 and that the city is estimated to have a population \(t\) years later of \(P(t) = 20000 \left(8^{t/20}\right)\). Use this formula to estimate the time for the size of the city to double.

Application (Real Life) Assessment Answer

The size of the city is estimated to double in \(\frac{20}{3} = 7 \frac{2}{3}\) years.
ALIGNED COURSE EXPECTATION C04

Solve exponential equations by use of logarithms, including applications.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following exponential equation. Give the exact solution.

\[ 4e^{-x^2} + 8 = 20 \]

Computational Assessment Answer

\[ x = 2 + \ln 3 \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

A company purchases two new cars, one for $32,000 and the other for $28,000. Each car depreciates according to the following formulas, respectively:

\[ V_1(t) = 32,000e^{-0.14t} \]

\[ V_2(t) = 28,000e^{-0.13t} \]

where \( V(t) \) represents the value of the car \( t \) years after it was purchased.

a) Which has the better depreciation rate?
b) When will the values of each car be the same?

Conceptual Assessment Answer

a) \( V_2(t) = 28,000e^{-0.13t} \) has the better depreciation rate since \(-0.13\) is greater than \(-0.14\).

b) Both cars will have the same value in 13.4 years.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The function

\[ L(t) = A \left( 1 - e^{-kt} \right) \]

can be used to approximate the amount \( L \) learned at time \( t \), where \( A \) represents the total amount learned at time \( t \) and \( k \) represents the learning rate. If Julie has 100 vocabulary words to learn and she has only learned 20 vocabulary words after studying for two hours, how many hours must Julie study before she learns 90 words?

Application (Real Life) Assessment Answer

It will take Julie approximately 10.3 hours to learn 90 words.
ALIGNED COURSE EXPECTATION C05

Evaluate logarithmic expressions by applying the Change of Base Formula.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Compute the following logarithmic expression to four decimal places using the Change of Base Formula and a calculator.

\[ \log_4(3) \]

Computational Assessment Answer

0.7925

CONCEPTUAL ASSESSMENT

Taxonomy: C

Use the Change of Base Formula to rewrite the following expression in terms of a base e logarithm.

\[ \log_4(5) \]

Conceptual Assessment Answer

\[ \frac{\ln(5)}{\ln(4)} \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

An engineer has solved an equation to determine the minimum thickness of glass required to meet building requirements. His solution is \( \log_2(6) \) inches. If the supplier can only supply glass with thicknesses in increments of \( \frac{1}{4} \)"—e.g., 0.25, 0.50, 0.75, 1.0, 1.25, ...—what is the minimum thickness the engineer could order?

Application (Real Life) Assessment Answer

2.75 inches
ALIGNED COURSE EXPECTATION C06

Condense a linear combination of simple logarithms into a single logarithm.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Write the expression as a single logarithm with a coefficient of 1:

$$\frac{1}{3}\log_2 x - 2\log_2 (y + 1) + 4\log_2 z.$$ 

Computational Assessment Answer

$$\log_2 z^{\frac{1}{3}} \cdot \frac{\sqrt[3]{x}}{(y + 1)^2}.$$ 

CONCEPTUAL ASSESSMENT

Taxonomy: C

The voltage across a loudspeaker changes from $V_1$ to $V_2$ when the volume is increased, and the decibel gain is

$$db = 20\log \frac{V_2}{V_1}.$$ 

a) Rewrite this equation so that it does not contain fractions.

b) Use the new equation to answer the question: If the voltage is increased to 10 times the original voltage, what is the decibel gain?

Conceptual Assessment Answer

a) $db = 20(\log V_2 - \log V_1)$ or $db = 20\log V_2 - 20\log V_1$ 

b) The resulting decibel gain will be 20 times the original.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The time that it takes for an investment, $P$, to grow to an amount, $A$, given an annual interest rate of $r$ and compounding $n$ times per year is given by the formula shown below. Solve this formula for $A$:

$$t = \frac{\ln(A/P)}{n\ln\left(1 + \frac{r}{n}\right)}$$

Application (Real Life) Assessment Answer

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$
ALIGNED COURSE EXPECTATION C07

Solve logarithmic equations by rewriting in exponential form, including applications.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following logarithmic equation: $\log_3(x + 7) - \log_3(x - 1) = 2$.

Computational Assessment Answer: 
$x = 2$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Determine the base, $b$, that will make the following equation true when $x = 259$:

$log_b(x^2 - 9) - log_b(x + 3) = b^3$.

Conceptual Assessment Answer

$b = 2$
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Crime scene analysts can approximate the time, $t$, in hours elapsed since death by measuring the body temperature, $T$, and using the following formula:

$$t = -10\ln \left( \frac{T - 70}{98.6 - 70} \right).$$

a) How long after death does it take a body to reach a temperature of 85° F?

b) How long after death does it take a body to reach a temperature of 80° F?

c) Determine the body temperature of a person who has been dead for 5 hours.

Application (Real Life) Assessment Answer

a) 6.45 hours

b) 10.51 hours

c) 87.35° F
**ALIGNED COURSE EXPECTATION C08**

Expand a logarithm into a linear combination of simple logarithms.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Expand each logarithm expression:

a) \( \log_b 3x^5 \)

b) \( \log_b \left( \frac{x^2}{36} \right) \)

**Computational Assessment Answer**

a) \( \log_b 3 + 5\log_b x \)

b) \( 2\log_b x - \log_b 36 \)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

A student claims the following is true:

\[ \log(u + v) = \log u + \log v = \log uv. \]

Explain how you would show that this is not true.

\[ \log(1 + 3) = \log(1) + \log(3), \]

\[ 0.6021 = 0 + 0.4771, \]

\[ 0.6021 \neq 0.4771. \]

**Conceptual Assessment Answer**

Use an example such as

\[ \log(1 + 3) = \log(1) + \log(3), \]

\[ 0.6021 = 0 + 0.4771, \]

\[ 0.6021 \neq 0.4771. \]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The formula for the period of simple harmonic motion, \( p \), is:

\[
p = 2\pi \sqrt{\frac{m}{k}},
\]

where \( m \) is the mass and \( k \) is the proportionality constant between stress and strain. Write \( \log p \) in expanded form using the properties of logarithms.

Application (Real Life) Assessment Answer

\[
\log p = \log 2 + \log \pi + \frac{1}{2} \log m - \frac{1}{2} \log k
\]
ALIGNED COURSE EXPECTATION C09

Solve logarithmic equations by applying a one-to-one property.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following logarithmic equation: $\log_2(x + 4) + \log_2(x - 2) = 4$.

Computational Assessment Answer

$x = 4$

CONCEPTUAL ASSESSMENT

Taxonomy: C

When you solve a logarithmic equation, why is it necessary to verify the solutions by substituting the proposed solutions into the original equation?

Conceptual Assessment Answer

After solving a logarithmic equation, it is necessary to verify the solutions to avoid extraneous solutions. The logarithm of a negative number is undefined. Any proposed solution that would result in taking the logarithm of a negative number is extraneous and should not be included as part of the final solution to the equation.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Alcohol slowly decays in the human body. If you have 13 mg. of alcohol in one drink and 3mg. is left in the bloodstream after 5 hours, how much alcohol will be left in your bloodstream after 12 hours?

Application (Real Life) Assessment Answer

0.386 mg.
**ALIGNED COURSE EXPECTATION C10**

Determine the domain and range of a logarithmic function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the domain and range of \( f(x) = \log_2 x + 5 \).

**Computational Assessment Answer**

Domain: \((0, \infty)\)

Range: \((-\infty, \infty)\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Given the function \( f(x) = \log_4 x \) and \( g(x) = \log_4 (x-3) \), what is the difference between the two functions with respect to domain and range?

**Conceptual Assessment Answer**

The range is the same for both functions. The domain of \( f(x) \) is \((0, \infty)\), and for \( g(x) \) it is \((3, \infty)\).
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Students in a mathematics class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model

\[ f(t) = 50 - 17 \log(t + 1), \ 0 \leq t \leq 12, \]

where \( t \) is the time in months. Using a calculator, determine the domain and range. Round your answers to four decimal places.

Application (Real Life) Assessment Answer

Domain: [0, 12]

Range: [50, 6.3959]
**Aligned Course Expectation C11**

Evaluate logarithmic expressions by applying properties of logarithms.

**Computational Assessment**

*Taxonomy: A*

Simplify each of the following expressions without using a calculator:

a) \( \log_{10} 0.01 \)

b) \( \log_{4} 32 \)

c) \( 5^{\log_{5} 40} \)

d) \( \ln e^{\log_{4} 4} \)

e) \( \log_{27} \left( \frac{\sqrt[3]{27}}{3} \right) \)

**Computational Assessment Answer**

a) -2

b) \( \frac{5}{2} \)

c) 40

d) \( \log 4 \)

e) \( -\frac{1}{12} \)

**Conceptual Assessment**

*Taxonomy: C*

Explain how you would simplify \( \log_{5} 81 \).
Conceptual Assessment Answer

Answers vary. Two possible answers are given below:

**Answer 1:** Rewrite 81 using a base of $\sqrt[4]{3}$. $81 = 3^4 = (\sqrt[3]{3})^3$. Therefore,

$$\log_{\sqrt[4]{3}} 81 = \log_{\sqrt[4]{3}} (\sqrt[3]{3})^8 = 4.$$

**Answer 2:** Use the change of base formula to rewrite

$$\log_{\sqrt[4]{3}} 81 = \frac{\log_3 81}{\log_3 \sqrt[4]{3}} = \frac{\log_3 3^4}{\log_3 3^{1/2}} = \frac{4}{1/2} = 8.$$

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy:** D

The formula

$$R = \log \left[ \frac{A}{A_0} \right]$$

can be used to determine the Richter magnitude of an earthquake, $R$, given seismograph readings for an earthquake, $A$, and a known seismograph reading of $A_0$. Use this formula to show how many times stronger a 7.0 earthquake is than an earthquake with a magnitude of 6.5.

**Application (Real Life) Assessment Answer**

A 7.0 earthquake is $10^{\frac{7}{2}} = 316$ times stronger than an earthquake with a 6.5 magnitude.
RATIONAL EXPRESSIONS & EQUATIONS

**ALIGNED COURSE EXPECTATION D01**

Graph a rational function by finding or creating the following: asymptotes, end and local behaviors, removable discontinuity, and key points.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Graph the following function:

\[ f(x) = \frac{x^2 + 6x - 7}{x^2 - x - 20}. \]

Computational Assessment Answer
CONCEPTUAL ASSESSMENT

Taxonomy: C

Determine which of the rational functions given below has the following feature(s):

x-intercepts 3 and 4; y-intercept 12; vertical asymptote x = 1; horizontal asymptote y = 1.

a) \( f(x) = \frac{x^2 - 7x + 12}{x - 1} \)

b) \( f(x) = \frac{x^2 - 7x + 12}{x^2 - 2x + 1} \)

c) \( f(x) = \frac{x^2 + 7x + 12}{x + 1} \)

d) \( f(x) = \frac{x^2 + 7x + 12}{x^2 + 2x + 1} \)

Conceptual Assessment Answer

b) \( f(x) = \frac{x^2 - 7x + 12}{x^2 - 2x + 1} \)

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Let the average number of vehicles arriving per minute at the gate of a water park be equal to \( k \), and let the average number of vehicles admitted by the park attendants be equal to \( r \). Then, the average waiting time, \( T \) (in minutes), for each vehicle arriving at the park is given by the rational function defined by the equation

\[ T(r) = \frac{-3r - k}{2r^2 - 2kr}, \]

where \( r > k \). It is known from experience that on Saturday afternoon, \( k = 25 \). Graph the function over a reasonable domain for this situation.
Application (Real Life) Assessment Answer

Answers may vary. The most important characteristic of the graph is that only $x > 25$ and $y > 0$ are represented.
**ALIGNED COURSE EXPECTATION D02**

Find vertical and horizontal asymptotes of rational functions.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the vertical and horizontal asymptotes for the following rational function:

$$f(x) = \frac{2x - 5}{x + 3}.$$

**Computational Assessment Answer**

Vertical asymptote at $x = -3$, and horizontal asymptote at $y = 2$.

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

In the rational function below, find a value for $a$ and $b$ so that the function has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 3$.

$$f(x) = \frac{ax + 5}{bx - 4}.$$

**Conceptual Assessment Answer**

$a = 6$ and $b = 2$
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A local company manufactures mountain bikes. The company estimates that the average cost of producing \( x \) bicycles is approximated by the function

\[
\bar{C}(x) = \frac{100x + 100,000}{x},
\]

where \( \bar{C}(x) \) is the average cost in dollars. What is the horizontal asymptote for this function and what does this say about the average cost of producing the bikes as the number of bikes produced increases?

Application (Real Life) Assessment Answer

The horizontal asymptote is located at \( y = 100 \). This indicates that the average cost of bicycles approaches $100 as more bicycles are produced.
ALIGNED COURSE EXPECTATION D03

Determine the domain and range of rational functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Determine the domain and range of the following rational function: \( f(x) = \frac{3x+2}{2x-1} \).

Computational Assessment Answer

Domain: \( (-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty) \)

Range: \( (-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty) \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Write a strategy for determining the domain of a rational function that is in the form \( f(x) = \frac{h(x)}{g(x)} \). (Note that \( h(x) \) and \( g(x) \) are both polynomials.)

Conceptual Assessment Answer

Step 1: Write the equation \( g(x) = 0 \).

Step 2: Solve this equation for \( x \).

Step 3: The domain of the function is the set of all real numbers with the solutions from step 2 excluded.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The state conservation commission introduces 100 bears into newly acquired land. The population $P(x)$ of bears is given by

$$P(x) = \frac{20(5+3x)}{1+0.04x} \quad x \geq 0,$$

where $x$ is time in years. Explain how the range of this rational function gives a good idea of what the population of bears will look like in the future.

Application (Real Life) Assessment Answer

The range on this function is $[100, 1500)$. This indicates that the population of bears will rise to close to 1,500 in the future.
ALIGNED COURSE EXPECTATION D04

Solve rational equations using algebraic, graphic, and tabular methods.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the equation \( \frac{5x + 2}{x + 1} + \frac{3}{x} = \frac{5}{x^2 + x} \).

Computational Assessment Answer

\[ x = \frac{-5 \pm \sqrt{65}}{10} \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Which equation has no restrictions and why?

a) \( \frac{x + 3}{x^2 + 4} = -\frac{2}{3} \)

b) \( \frac{x + 3}{x^2 - 4} = -\frac{2}{3} \)

Conceptual Assessment Answer

Answer choice (a) because there is no real number value that would cause the denominator \( x^2 + 4 \) to equal zero.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The simple interest rate on a savings account is 2% higher than the simple interest rate on a checking account. After 1 year the interest earned on a deposit in the checking account is $30 and the interest earned on a deposit in the savings account is $60. Find the interest rate for each account if the same amount of money was deposited into each account.

Application (Real Life) Assessment Answer

Using the equation

$$\frac{30}{r} = \frac{60}{r+2},$$

where \( r \) represents the interest rate of the checking account, the interest rate is 2% for the checking account and 4% for the savings account.
RADICAL EXPRESSIONS & EQUATIONS

● ALIGNED COURSE EXPECTATION E01

Find the domain and range of radical functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Determine the domain and range of the radical function \( f(x) = \sqrt{15 - 3x} - 2 \).

Computational Assessment Answer

Domain: \((-\infty, 5]\)

Range: \([-2, \infty)\)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Consider the function \( f(x) = \sqrt{x} \). Given that a real number \( a \) is in the domain of \( f(x) = \sqrt{x} \), what can be said about \( a \)?

Conceptual Assessment Answer

\( a \geq 0 \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The distance to the horizon, \( d \), that you can see from a given height, \( h \), is given by the equation \( d = 1.2\sqrt{h} \). Find the domain of \( d \) as a function of \( h \).

Application (Real Life) Assessment Answer

\[ h \geq 0 \]
**ALIGNED COURSE EXPECTATION E02**

Find the inverses of radical functions.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Find the inverse of the function $f(x) = \sqrt[3]{2x - 1}$.

**Computational Assessment Answer**

$$f^{-1}(x) = \frac{1}{2}x^3 + \frac{1}{2}$$

**CONCEPTUAL ASSESSMENT**

Taxonomy: B

Determine the domain and range of $f(x) = \sqrt{2 - x} + 5$. Then, find the inverse of $f(x)$ as well as the domain and range of the inverse.

**Conceptual Assessment Answer**

Domain of $f(x): (-\infty, 2]$  
Range of $f(x): [5, \infty)$  
Domain of $f^{-1}(x): [5, \infty)$  
Range of $f^{-1}(x): (-\infty, 2]$
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The function \( v(d) = \sqrt{19.62d} \) gives the instantaneous velocity in feet per second of a freely falling object that has traveled distance \( d \) in feet. Find \( d(v) \), the inverse of \( v(d) \), and describe in practical terms the meanings of \( d(19.81) = 20 \).

Application (Real Life) Assessment Answer

\[ v(d) = \frac{d^2}{19.62} \]

When the object is traveling at a velocity 19.81 ft./sec., the object will have fallen 20 ft.
ALIGNED COURSE EXPECTATION E03

Solve radical equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following radical equation for $x$: $\sqrt{2x+1} + \sqrt{x-3} = 4$.

Computational Assessment Answer

$x = 4$ ($x = 84$ is extraneous)

CONCEPTUAL ASSESSMENT

Taxonomy: B

Explain why squaring both sides of an equation can produce extraneous solutions. Give a simple example to illustrate your explanation.

Conceptual Assessment Answer

Real numbers which differ only in sign, e.g., $+4$ and $-4$, will have the same square, e.g., $16$. Values of $x$ that make both sides of an equation differ only in sign are not solutions to that equation, but they will be solutions to the equation produced by squaring both sides.

For example, the equation $2x = 8$ has the unique solution $x = 4$. The equation obtained by squaring both sides is $4x^2 = 64$, which has solutions $x = -4$ and $x = 4$. Squaring both sides produced the extraneous solution $x = -4$. 
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The function $n = 0.2\sqrt[3]{d^3}$ models the number of days (24 hour cycles), $n$, required for a planet to orbit the Sun, where $d$ is the average distance between the planet and the Sun (in gigameters). Given that it takes Mercury 88 days to orbit the Sun, find the average distance between Mercury and the Sun (round your solution to the nearest gigameter).

Application (Real Life) Assessment Answer

About 58 gigameters.
POLYNOMIAL EXPRESSIONS & EQUATIONS

● ALIGNED COURSE EXPECTATION F01

Evaluate polynomial functions by substitution or by synthetic division and the Remainder Theorem.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use synthetic division and the Remainder Theorem to evaluate the polynomial

\[ f(x) = 4x^4 - 2x^3 + x^2 + 4 \] at \( x = -1 \).

Computational Assessment Answer

11

CONCEPTUAL ASSESSMENT

Taxonomy: C

If \((x - c)\) is a factor of the polynomial \(p(x)\), what is \(p(c)\)?

Conceptual Assessment Answer

0

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

Let \(f(x) = x^3 - 4x + c\). Use synthetic division to find the value of \(c\) for which \(f(4) = 70\).

Application (Real Life) Assessment Answer

22
ALIGNED COURSE EXPECTATION F02

Factor a polynomial of degree 3 or higher using the Factor Theorem.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use the Factor Theorem to factor the polynomial expression $3x^3 - 19x^2 + 30x - 8$ completely.

Computational Assessment Answer

Factored form: $(3x - 1)(x - 2)(x - 4)$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Given that $(x - c)$ is a factor of the polynomial $p(x)$, what is the remainder of $p(x) ÷ (x - c)$?

Conceptual Assessment Answer

0

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

A cubic function $f(x)$ has roots of $-2$ and $2 + i$. Find the function.

Application (Real Life) Assessment Answer

$f(x) = x^3 - 2x^2 - 5x + 6$
**ALIGNED COURSE EXPECTATION F03**

Apply Rational Root Theorem, Factor Theorem, and synthetic division to find all roots (zeros) of polynomials.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Consider the polynomial function: \( f(x) = x^3 + 3x^2 + 9x - 13 \).

a) Use the Rational Root Theorem to list all possible rational zeros of the polynomial function.

b) Use synthetic division to identify a rational zero of the polynomial function.

c) Use the results above to find all remaining zeros of the polynomial function.

**Computational Assessment Answer**

a) possible rational zeros: \{-13, -1, 1, 13\}

b) 1

c) remaining zeros: 1, \(-2 - 3i, -2 + 3i\)

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

The function \( f(x) = x^3 + 3x^2 + 9x - 13 \) has at least one complex root. How many more distinct roots does it have? Of what type?

**Conceptual Assessment Answer**

This function has two more distinct roots: one complex and one real.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A rectangular prism has a volume of 32 cubic inches. If the length is 3 inches more than the width, and the height is 4 inches more than the length, find the dimensions of the prism.

Application (Real Life) Assessment Answer

The width is 1 inch, the length is 4 inches, and the height is 8 inches.
ALIGNED COURSE EXPECTATION F04

Identify the \( x \)-intercepts (if applicable) and \( y \)-intercept of polynomial functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Give the \( x \)-intercepts and \( y \)-intercept of the graph of \( f(x) = x^3 + 3x^2 - 4x - 12 \).

Computational Assessment Answer

The \( x \)-intercepts are: \((-3, 0), (-2, 0), \) and \((2, 0)\). The \( y \)-intercept is \((0, -12)\).

CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose a third-degree polynomial function has one complex root. How many \( x \)-intercepts does its graph have? How many \( y \)-intercepts?

Conceptual Assessment Answer

One \( x \)-intercept and one \( y \)-intercept.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose that at \( t = 0 \), an arrow is fired straight up into the air at an initial speed of 32 feet/second from an initial height of 48 feet and strikes the ground 3 seconds later, at \( t = 3 \). Let \( h(t) \) be the height of the arrow as a function of time, \( t \). Give the positive \( x \)-intercept and the \( y \)-intercept of \( h(t) \).

Application (Real Life) Assessment Answer

\( x \)-intercept: \((3, 0)\); \( y \)-intercept: \((0, 32)\)
ALIGNED COURSE EXPECTATION F05

Determine the equation of a polynomial function from given roots (real or complex), including in an application.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

A cubic function, \( f(x) \), has roots of \(-2\) and \(2+1\). If \( f(0) = 6 \), find the function.

Computational Assessment Answer

\[ f(x) = x^3 - 2x^2 - 5x + 6 \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose a cubic polynomial function has the roots \(a + bi\). What do you know about the other two roots?

Conceptual Assessment Answer

One is real and one is complex. The complex root is \(-a - bi\).

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Let \(x\) be the width of a rectangular box whose length is twice its width. If its height is 4 inches greater than its width, write an expression representing the volume of the box.

Application (Real Life) Assessment Answer

\[ V(x) = 2x^3 + 8x^2 \]
ALIGNED COURSE EXPECTATION F06
Describe end behavior of a polynomial function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Describe the end-behavior of the polynomial function \( f(x) = 4 - x - 3x^4 \).

Computational Assessment Answer

As \( x \to -\infty \), \( f(x) \to -\infty \); and as \( x \to \infty \), \( f(x) \to -\infty \).

CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose the end-behavior of a polynomial function is described as follows: as \( x \to -\infty \), \( f(x) \to \infty \); and as \( x \to \infty \), \( f(x) \to -\infty \). What can be determined about the degree of the polynomial function and its lead coefficient?

Conceptual Assessment Answer

The degree is odd and the lead coefficient is negative.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A nuclear technician analyzes a nuclear reactor to produce a polynomial model of the reactor’s temperature, \( T(t) \), as a function of time, \( t \). Suppose \( T(t) \) is a fourth-degree polynomial function with a positive lead coefficient. What does this model predict will happen to the temperature in the reactor far into the future?

Application (Real Life) Assessment Answer

The model predicts that the temperature in the reactor will continue to increase to infinity.
ALIGNED COURSE EXPECTATION F07

Solve a polynomial inequality of degree two or higher.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the polynomial inequality $x^3 - x^2 - 4x \leq -4$.

Computational Assessment Answer

The solution set is $\{x \mid 1 \leq x \leq 2 \text{ or } x \leq -2\}$ or $(-\infty, -2] \cup [1, 2]$.

CONCEPTUAL ASSESSMENT

Taxonomy: C

Consider the graph of $f(x)$ below:

Use the graph to solve $(-\infty, -2] \cup [1, 2]$.
Conceptual Assessment Answer

The solution set is $(-\infty, -2] \cup [1, 2]$.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

For safety purposes, fireworks are timed to explode above a height of 40 meters. Suppose a firework is fired straight up from the ground. Its height is given by $h(t) = -4.9t^2 + 98t$.

To the nearest tenth of a second, for what range of times can the firework be safely detonated?

Application (Real Life) Assessment Answer

The firework may be safely detonated between 0.4 seconds and 19.6 seconds after launch.
ALIGNED COURSE EXPECTATION F08

Solve a quadratic equation by completing the square.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the quadratic equation $2x^2 - 10x = 37$ for $x$ by completing the square.

Computational Assessment Answer

$x = \frac{5 - 3\sqrt{11}}{2}$ and $x = \frac{5 + 3\sqrt{11}}{2}$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Use completing the square to solve the quadratic equation $x^2 - mx = 1$ for $x$.

Conceptual Assessment Answer

$x = \frac{m - \sqrt{m^2 + 4}}{2}$ and $x = \frac{m + \sqrt{m^2 + 4}}{2}$

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The profit, $P(x)$, from selling $x$ T-shirts has been determined to be $P(x) = -2x^2 + 28x - 26$. Use completing the square to find the values of $x$ for which $P(x) = 0$. These values are called the “break even points” for the profit function.

Application (Real Life) Assessment Answer

The break even points are 1 and 13.
**ALIGNED COURSE EXPECTATION F09**

Solve quadratic equations and applications (projectile motion, geometric area, Pythagorean Theorem, etc.) using the algebraic method of the Quadratic Formula.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the quadratic equation $9x^2 + 9 = 12x$ for $x$ by using the Quadratic Formula.

Computational Assessment Answer

$$x = \frac{2 - \sqrt{5}}{3} \text{ and } x = \frac{2 + \sqrt{5}}{3}$$

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

Use the Quadratic Formula to solve the quadratic equation $x^2 - mx = 1$ for $x$.

Conceptual Assessment Answer

$$x = \frac{m - \sqrt{m^2 + 4}}{2} \text{ and } x = \frac{m + \sqrt{m^2 + 4}}{2}$$

**APPLICATION (REAL LIFE) ASSESSMENT**

Taxonomy: D

The profit, $P(x)$, from selling $x$ T-shirts has been determined to be $P(x) = -2x^2 + 28x - 26$. Use the Quadratic Formula to find $x$ for which $P(x) = 0$. These values are called the “break even points” for the profit function.

Application (Real Life) Assessment Answer

The break even points are 1 and 13.
ALIGNED COURSE EXPECTATION F10

State the maximum number of turning points for a given polynomial function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A
Use the degree to give the maximum number of turns of the polynomial function

\[ f(x) = -2x^3 + 28x^2 - 26. \]

Computational Assessment Answer

2

CONCEPTUAL ASSESSMENT

Taxonomy: C
Give the maximum number of terms of an \( n \)-degree polynomial function.

Conceptual Assessment Answer

\( n - 1 \)

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C
Suppose a polynomial function has three turns. What is the greatest number of distinct, real zeros the polynomial can possess? What is the least?

Application (Real Life) Assessment Answer

The polynomial function can possess at most four distinct, real zeros. It could possibly possess no real zeros.
**ALIGNED COURSE EXPECTATION F11**

Use the multiplicity of a zero to determine the behavior of the graph of a polynomial function at each $x$-intercept.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Consider the polynomial function $f(x) = x^3(x - 2)^4(3x + 1)^3$. Find the $x$-intercepts of the graph of this polynomial function. Use the multiplicities of each zero to determine at each $x$-intercept whether the graph crosses the $x$-axis or touches the $x$-axis and turns.

**Computational Assessment Answer**

$x$-intercepts: $\left(-\frac{1}{3}, 0\right), (0, 0), (2, 0)$

At $\left(-\frac{1}{3}, 0\right)$, the graph crosses the $x$-axis. At $(0, 0)$ and $(2, 0)$, the graph touches the $x$-axis and turns.

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Suppose $c$ is a real zero of the polynomial function $f(x)$ with multiplicity $m$. Given that the graph of $f(x)$ crosses the $x$-axis at $(c, 0)$, what do you know about $m$?

**Conceptual Assessment Answer**

$m$ must be odd
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: C

Suppose the graph of \( f(x) \) crosses the \( x \)-axis at \( (c, 0) \). A new polynomial, \( g(x) \), is formed by multiplying \( f(x) \) by \( p(x) \)—that is, \( g(x) = p(x)f(x) \). If the graph of \( g(x) \) touches the \( x \)-axis and turns at \( (c, 0) \), what can be said about the multiplicity of \( c \) as a zero of \( p(x) \)?

Application (Real Life) Assessment Answer

\( c \) has odd multiplicity as a zero of \( p(x) \)
**ALIGNED COURSE EXPECTATION F12**

Graph a polynomial function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Sketch a graph of \( f(x) = -\frac{1}{4}x^5 + x^4 + \frac{3}{4}x^3 - \frac{9}{2}x^2 \).

**Computational Assessment Answer**
CONCEPTUAL ASSESSMENT

Taxonomy: C

Given the graph of \( f(x) \) below, determine the effect of multiplying each of the following functions by \( x \).

a) Number of \( x \)-intercepts

b) Number of turns

c) End behavior

Conceptual Assessment Answer

a) The number of \( x \)-intercepts would stay the same.

b) The number of turns would increase by two.

c) The end behavior would change to: as \( x \to -\infty, f(x) \to \infty \); and as \( x \to \infty, f(x) \to \infty \).

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
CONIC SECTIONS

ALIGNED COURSE EXPECTATION G01

Given the equation of a circle, identify the center and radius of the circle and sketch the graph.

COMPUTATIONAL ASSESSMENT

Taxonomy: A
Find the center and radius of a circle from the following equation:

\[(x+2)^2 + (y-3)^2 = 16.\]

Computational Assessment Answer
Center: (-2, 3)
Radius: 4

CONCEPTUAL ASSESSMENT

Taxonomy: A
Compare the graphs of the two equations below:

\[(x-3)^2 + (y+4)^2 = 9\]
\[(x+3)^2 + (y-4)^2 = 9\]

How are the graphs the same and how are they different?

Conceptual Assessment Answer
Both graphs are circles with a radius of 3. The first graph has a center of \((3, -4)\), while the second graph has a center of \((-3, 4)\).
**ALIGNED COURSE EXPECTATION G02**

Determine the equation of the circle given geometric information.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

If the diameter of a circle has endpoints \((-8, -3)\) and \((2, 1)\), then write the equation of the circle.

**Computational Assessment Answer**

\[(x + 3)^2 + (y + 1)^2 = 29\]

**CONCEPTUAL ASSESSMENT**

**Taxonomy: A**

Students are told to write the equation of a circle and graph it. If Melissa graphs the equation \((x + 3)^2 + (y - 2)^2 = 9\) and Mike graphs the equation \((x + 3)^2 + (y - 2)^2 = 36\), how do their circles compare?

**Conceptual Assessment Answer**

Both circles have a center at \((-3, 2)\), and the radius of Mike’s circle is twice the length of the radius of Melissa’s circle.

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: D**

If a city is located at \((-2, 3)\) and the epicenter of an earthquake is located 5 miles away, write the equation of a circle representing possible locations of the earthquake’s epicenter.

**Application (Real Life) Assessment Answer**

\[(x + 2)^2 + (y - 3)^2 = 25\]
ASSESSMENTS FOR ENTRY-LEVEL Precalculus

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FUNCTIONS & MISCELLANY

**ALIGNED COURSE EXPECTATION A01**

Perform basic operations involving complex numbers.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Simplify the following expressions to a single complex number of the form $a+bi$:

a) $(3+7i)+(-9+2i)$

b) $(5-4i)-(7-11i)$

c) $(3+i)(5-2i)$

d) $\frac{3+5i}{2-3i}$

**Computational Assessment Answer**

a) $-6+9i$

b) $-2+7i$

c) $17-i$

d) $\frac{9}{13}+\frac{19}{13}i$
CONCEPTUAL ASSESSMENT

Taxonomy: C

The complex number $-1 - i$ is a solution of the equation $x^2 + 2x + 2 = 0$.

a) Write down a second solution of the equation.

b) Verify/show that your answer of part (a) is a solution of the equation.

Conceptual Assessment Answer

a) Complex solutions of polynomial equations occur in complex conjugate pairs. Therefore, if $-1 - i$ is a solution, then a second solution must be $-1 + i$.

b) Substituting $-1 + i$ for $x$ in the equation above verifies it is a solution.

\[
(-1+i)^2 + 2(-1+i) + 2 = (-1+i)(-1+i) + 2(-1+i) + 2
\]
\[
= 1 - i + i^2 - 2 + 2i + 2
\]
\[
= 1 - 2i + (-1) - 2 + 2i + 2
\]
\[
= 1 - 2i - 1 - 2i + 2
\]
\[
= 1 - 2 + 2 - 2i + 2i
\]
\[
= 0
\]

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
ALIGNED COURSE EXPECTATION A02

Simplify algebraic expressions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Simplify the following expressions, writing the final result using only positive exponents.

a) \((4x^6)(5x^3)\)

b) \(7a^5 (2a^3)^2\)

c) \(\left(\frac{x^8}{x^2}\right)\left(\frac{y^3}{y^4}\right)\)

Computational Assessment Answer

a) \(20x^9\)

b) \(28a^{11}\)

c) \(x^5y^7\)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Show why \((x+y)^2 = x^2 + y^2\) is incorrect (if neither \(x\) nor \(y\) is zero).

Conceptual Assessment Answer

\[
(x+y)^2 = (x+y)(x+y) \\
= x(x+y) + y(x+y) \\
= x^2 + xy + yx + y^2 \\
= x^2 + 2xy + y^2
\]
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The current world population is approximately 7.2 billion, and the surface area of the earth is approximately 197 million square miles. If the world’s population was evenly distributed over the earth’s surface, approximately how many people would there be per square mile?

Application (Real Life) Assessment Answer

$$7,200,000,000 = 7.2 \times 10^9$$

$$197,000,000 = 1.97 \times 10^8$$

To calculate the number of people per square mile, divide the world's population by the number of square miles.

$$\frac{7.2 \times 10^9}{1.97 \times 10^8} = \frac{7.2 \times 10^9}{1.97 \times 10^8} \approx 3.65 \times 10^1 \approx 36.5 \text{ people per sq. mile}$$

So, there are approximately 36–37 people per square mile.
ALIGNED COURSE EXPECTATION A03

Determine the domain and range of a relation given a table, graph, or equation.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Find the domain and range of the following functions, writing the answer using interval notation.

a)

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>-17</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

b)
c) \( g(x) = \frac{3x-4}{x+5} \)

**Computational Assessment Answer**

a) Domain: \( \{-1,2,3,5,8\} \)

Range: \( \{5,9,23,-17,4\} \)

b) Domain: \([-8,7]\)

Range: \([-3,7]\)

c) Domain: \(x \neq -5\) or \((-\infty,-5) \cup (-5,\infty)\)

Range: \(x \neq 3\) or \((-\infty,3) \cup (3,\infty)\)

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Write down a function whose domain is:

a) \( f(x) = \frac{x+8}{x-3} \)

b) \( g(x) = \sqrt{x-4} \)

**Conceptual Assessment Answer**

a) \((-\infty,3) \cup (3,\infty)\)

b) \(4 \leq x\)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

A tank holds 100 gallons of water and the water is leaking out of the tank. The amount/volume, \( V \), of water in the tank after \( t \) minutes, according to Torricelli’s Law, is given by the function

\[
V(t) = 100 \left(1 - \frac{t}{30}\right)^2.
\]

According to this function, what is the time interval over which the tank will be emptied?

Application (Real Life) Assessment Answer

The domain of the function is the time interval over which the volume goes from 100 to 0. \( V = 0 \) when the quantity inside the parentheses is zero—that is, when

\[
1 - \frac{t}{30} = 0, \text{ or } t = 30.
\]

So the domain of the function is \([0, 30]\), or \(0 \leq t \leq 30\), and the tank is emptied after 30 minutes.
ALIGNED COURSE EXPECTATION A04

Find the composition of two functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

a) Functions \( f \) and \( g \) are given by \( f(x) = 2x^2 + x \) and \( g(x) = 5 - 2x \).

Find the value of \((f \circ g)(3)\) and \((g \circ f)(5)\).

b) Functions \( g \) and \( h \) are given by \( g(x) = 3x + 2 \) and \( h(x) = x^2 - x \).

Find \((g \circ h)(x)\) and \((h \circ g)(x)\).

c) Consider the following values for functions \( f \) and \( g \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td>9</td>
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<td>1</td>
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<td>7</td>
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<td>3</td>
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<tr>
<td>10</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Find the value of \((f \circ g)(7)\) and \((g \circ f)(10)\).
d) The graphs of functions $f$ and $g$ are shown below.

Find the value of $(g \circ f)(1)$ and $(f \circ g)(3)$.

**Computational Assessment Answer**

\[
(f \circ g)(3) = f(g(3)) = f(-1) = 2(-1)^2 + (-1) = 2 - 1 = 1
\]

\[
(g \circ f)(5) = g(f(5)) = g(21) = 5 - 2(21) = 5 - 42 = -37
\]

\[
(g \circ h)(x) = g(h(x)) = g(x^2 - x) = 3(x^2 - x) + 2 = 3x^2 - 3x + 2
\]

\[
(h \circ g)(x) = h(g(x)) = h(3x + 2) = (3x + 2)^2 - (3x + 2)
\]

\[
= 9x^2 + 6x + 6x + 4 - 3x - 2 = 9x^2 + 9x + 2
\]
\[ (f \circ g)(7) = f(g(7)) \quad (g \circ f)(10) = g(f(10)) \]

\[ c) \quad f(3) = 7 \quad \text{and} \quad g(3) = -2 \]

\[ (g \circ f)(11) = g(f(11)) \quad (f \circ g)(3) = f(g(3)) \]

\[ d) \quad g(6) = 9 \quad \text{and} \quad f(-1) = 4 \]

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Find two functions \( f \) and \( g \) for which \( (f \circ g)(x) = \sqrt{x^2 + 9} \).

**Conceptual Assessment Answer**

Let \( g(x) = x^2 + 9 \) and \( f(x) = \sqrt{x} \).

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: D**

After Christmas, a store owner offers a 20% discount followed by a $50 rebate. Write down two functions, \( d \) for the discount and \( r \) for the rebate, so that the composition of these two functions yields the correct final price. Then use this composition function to find the final price for an item originally priced at $200.

**Application (Real Life) Assessment Answer**

Let \( d(x) = 0.80x \) and \( r(x) = x - 50 \).

\[ (r \circ d)(x) = r(d(x)) \]

Then,

\[ = r(0.80x) \]

\[ = 0.80x - 50 \]

and \( (r \circ d)(200) = 0.80(200) - 50 = 110 \), so the item has a final selling price of $110.
ALIGNED COURSE EXPECTATION A05

Identify the parent function (square, cube, square root, cube root, absolute value, reciprocal, exponential/logarithmic) and transformations for a given transformed function and use the transformations to graph the functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

a) Sketch the graph of \( y = 10 + \sqrt{x - 5} \) from the square root function graph \( y = \sqrt{x} \).

b) Sketch the graph of \( y = \left( \frac{1}{2} \right) (x - 2)^3 - 3 \) from the cubing function graph \( y = x^3 \).

Computational Assessment Answer

a) [Graph of functions]

\( y = 10 + \sqrt{x - 5} \)

\( y = \sqrt{x} \)

\( y = \sqrt{x - 5} \)
CONCEPTUAL ASSESSMENT

Taxonomy: A

a) Find the function whose graph is the cube root function that is first shifted four units to the left, followed by a vertical shift downward of seven units.

b) Find the function whose graph is the cube function that is first shifted two units to the right, shrunk by a factor of one-half, then raised vertically three units.

c) Describe, in words, how the graph of \( y = 5|x−8|+10 \) is derived from the graph of \( y = |x| \).

d) Identify the parent function of \( g(x) = 3\log(x+7) \).

e) Identify the parent function of \( h(x) = -5e^{x−2} \).

f) Identify the parent function of \( r(x) = \frac{3}{x+10} \).
Conceptual Assessment Answer

a) \( y = \sqrt{x+4} - 7 \)

b) \( y = \frac{1}{2} (x-2)^3 + 3 \)

c) **step 1:** shift the graph of \( y = |x| \), the absolute value function graph, eight units to the right

**step 2:** stretch this graph vertically by a factor of five

**step 3:** raise this graph vertically by 10 units

d) The parent function is \( f(x) = \log(x) \).

e) The parent function is \( f(x) = e^x \).

f) The parent function is \( f(x) = \frac{1}{x} \).

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
ALIGNED COURSE EXPECTATION A06

Apply polynomial, logarithmic, and exponential functions to model real-life data.

COMPUTATIONAL ASSESSMENT

No assessment.

CONCEPTUAL ASSESSMENT

No assessment.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

a) A farmer has 400 feet of fence with which to construct a rectangular enclosure for some of his animals. Since the farmer intends to use a straight stretch of river for one side of the enclosure, fencing will only be required for three sides. Find a function for the area, \( A \), of the enclosure in terms of only \( x \), where \( x \) is the length of a side perpendicular to the river. Then determine the dimensions of the largest possible enclosure and its area.

b) Carbon dating is a method that has been developed to determine the age of certain old objects. One formulation is

\[
A \approx -8,300 \ln \left( \frac{D}{D_o} \right)
\]

where \( A \) is the age in years, \( D_o \) is the original amount of carbon-14, and \( D \) is the amount of remaining carbon-14. How old is a piece of statuary found at an archeological site if 65% of the original carbon-14 remains?

c) When a mild nonsteroidal anti-inflammatory drug (NSAID) is administered, the number of milligrams remaining in the patient's bloodstream after \( t \) hours is given by the function \( R(t) = 100e^{-0.15t} \). How many milligrams did the patient take initially, and how long will it take for one-half of this initial amount to be removed from their bloodstream?
Application (Real Life) Assessment Answer

a) If \( x \) is the length of a side perpendicular to the river, then the area, \( A \), of the enclosure may be expressed as \( A(x) = x(400 - 2x) = 400x - 2x^2 \). The dimensions of the largest possible enclosure are 100 ft. \( \times \) 200 ft., where the two sides of fence that are perpendicular to the river are 100 ft. each. The area of this enclosure is 20,000 ft.\(^2\).

b) \[
\frac{D}{D_0} = \frac{0.65D}{D_0} = 0.65,
\]
so \( A = -8,300 \ln(0.65) \approx 3,575 \), which means the artifact is approximately 3,575 years old.

c) The initial amount of NSAID that is administered is \( R(0) = 100e^{-0.15(0)} = 100e^0 = 100 \), so 100 milligrams. When one-half of the initial amount has been removed from the bloodstream, one-half remains, so \( R(t) = 50 \) for \( t \approx 46.2 \). This means it takes about 46.2 hours for one-half of the initial amount to be removed from the bloodstream.
ALIGNED COURSE EXPECTATION A07

Algebraically, determine the inverse of a function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

a) Find the inverse function $f^{-1}$ for the function $f(x) = 3x - 7$.

b) Find the inverse function $g^{-1}$ for the function $g(x) = \frac{2x + 1}{x - 2}$.

Computational Assessment Answer

a) $f^{-1}(x) = \frac{x + 7}{3}$ or $f^{-1}(x) = \frac{1}{3}x + \frac{7}{3}$

b) $g^{-1}(x) = \frac{2x + 1}{x - 2}$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Find a function $f$ for which the domain of the inverse function is $(-\infty, 4) \cup (4, \infty)$.

Conceptual Assessment Answer

Start with an inverse function formula of $x = \frac{3y + 1}{y - 4}$ for which $y \neq 4$.

Solving for $y$, you get $y = \frac{4x + 1}{x - 3}$, so the desired function is $f(x) = \frac{4x + 1}{x - 3}$. 
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: B

A storage tank holds 200 gallons of water. Water drains out of the tank (according to Torricelli’s Law) through a leak in the bottom in such a way that the volume of water, $V$, in the tank at time $t$ minutes is given by

$$V(t) = 200 \left(1 - \frac{t}{50}\right)^2$$

for $0 \leq t \leq 50$. Use the inverse function to determine the time it takes for 128 gallons to leak from the tank.

**Application (Real Life) Assessment Answer**

If 128 gallons have leaked from the tank, then the volume of water remaining in the tank is $200 - 128 = 72$, which means the volume, $V$, of water in the tank is 72 gallons.

The inverse function, $V$, “formula” is given by

$$t = 50 \left(1 - \frac{V}{200}\right), \text{ so } V^{-1}(72) = 50 \left(1 - \frac{72}{200}\right) = 20.$$

Therefore, it takes 20 minutes for 128 gallons to leak from the tank.
ALIGNED COURSE EXPECTATION A08

Evaluate and graph a given piecewise function.

COMPUTATIONAL ASSESSMENT

Taxonomy: C
Evaluate the function at \( x = 2 \):

\[
f(x) = \begin{cases} 
-x - 3 & \text{if } 0 \leq x \leq 3 \\
4x + 1 & \text{if } 3 < x < 9
\end{cases}
\]

Computational Assessment Answer

\[
f(2) = -(2) - 3 = -5
\]

CONCEPTUAL ASSESSMENT

Taxonomy: C
Graph the function:

\[
f(x) = \begin{cases} 
-x - 3 & \text{if } 0 \leq x \leq 3 \\
4x + 1 & \text{if } 3 < x < 9
\end{cases}
\]
Conceptual Assessment Answer

Application (Real Life) Assessment Answer

Application (Real Life) Assessment

Taxonomy: B

Your income tax rate is 0% on the first $10,000 earned, then 10% on the next $90,000, then 15% on all money earned above $100,000. This is modeled in the piece-wise function below:

\[
T(x) = 
\begin{cases} 
0 & \text{if } 0 \leq x \leq 10,000 \\
.1x - 1000 & \text{if } 10,000 \leq x \leq 100,000 \\
.15x - 6000 & \text{if } 100,000 \leq x 
\end{cases}
\]

Use \( T(x) \) to find your income tax if you made $119,000.

Application (Real Life) Assessment Answer

\[
T(119000) = .15(119000) - 6000 = 11850
\]

The tax on $119,000 income is $11,850.
ALIGNED COURSE EXPECTATION A09

Determine symmetry of a relation given the graph or equation.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Determine whether each of the functions given below is:

1) Symmetric about the $y$-axis
2) Symmetric about the origin
3) Symmetric about a vertical axis of symmetry
4) None of the above

a) $f(x) = 2x^2 - 5x + 11$

b) $g(x) = x^5 - 12$

c) $f(x) = x\sqrt{x^4 + 5}$

d) $j(x) = \frac{\sqrt[3]{x}}{x}$

Computational Assessment Answer

1) a
2) d
3) b
4) a and c
CONCEPTUAL ASSESSMENT

Taxonomy: C

Suppose \( f \) is an even function—that is, \( f(-x) = f(x) \) for all \( x \) in the domain of \( f \). If \((-2, 17)\) is a point in the graph of \( f \), give another point in the graph.

Conceptual Assessment Answer

\((2, 17)\) is another point in the graph

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Adam is a pyrotechnician working on a fireworks display for an upcoming concert. He has a new mortar launcher that fires mortars along a parabolic trajectory to a height of 140 ft. For the fireworks he plans to use, the optimal detonation height is above 110 ft. If it takes the fireworks two seconds to travel from 110 ft. to 140 ft., how many seconds does Adam have to detonate his fireworks at or above 110 ft.?

Application (Real Life) Assessment Answer

4 seconds
**SYSTEMS**

**ALIGNED COURSE EXPECTATION B01**

Solve a system of linear equations.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Solve the following system of equations using Gauss-Jordan Elimination:

\[
\begin{align*}
3x - 2y + z &= -5 \\
x + 3y - 2z &= 14 \\
x + y - z &= 6
\end{align*}
\]

*Computational Answer*

\((1, 3, -2)\)

**CONCEPTUAL ASSESSMENT**

Taxonomy: C

As part of her solution using Gauss-Jordan Elimination, Mary has the following matrix:

\[
\begin{bmatrix}
1 & 1 & 2 & | & 3 \\
0 & 2 & 3 & | & 4 \\
0 & 0 & 2 & | & 5
\end{bmatrix}
\]

To get a 1 in the pivot position for column two, Mary decides to interchange rows one and two. Explain why this is not an appropriate step.

*Conceptual Answer*

It will change column one, which is already in the correct form.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Hill Country Supplies makes three types of tents: two-person models, three-person models, and four-person models, which cost $75, $100, and $125, respectively. A two-person tent provides 3 cubic yards of space, a three-person tent provides 5 cu. yds. of space, and a four-person tent provides 8 cu. yds. of space. An organization that takes kids camping ordered enough tents to hold 166 people and provide 289 cu. yds. of space. If the cost was $5,575, how many of each type of tent were ordered?

Application (Real Life) Assessment Answer

Two-person tents: 19
Three-person tents: 24
Four-person tents: 14
EXEMPLARY & LOGARITHMIC EXPRESSIONS & EQUATIONS

ALIGNED COURSE EXPECTATION C01

Solve an exponential equation (including real world situations).

COMPUTATIONAL ASSESSMENT

Taxonomy: A

If a termite population can be approximated by the function \( P(t) = 700,000e^t \), where \( t \) represents the number of weeks after an initial approximation was taken, then determine the number of weeks until the termite population exceeds 3 million.

Computational Assessment Answer

\[ t = 3 \]

CONCEPTUAL ASSESSMENT

Taxonomy: C

Let \( A \) represent the mass of Fermium-253 (\( {}^{253}\text{Fe} \)) (in grams), whose half-life is three days. The quantity of Fermium-253 present after \( t \) days is

\[ A = 4 \times \left( \frac{1}{2} \right)^{t/3} \]

Use rules of exponents in order to determine how many days it will take until one-half of a gram of Fermium remains.

Conceptual Assessment Answer

Nine days
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Steve is given the opportunity to refinance his house, valued at $150,000, at either 3% annual interest, compounded monthly, for 30 years, or at 2.875%, compounded monthly, for 15 years. Assume that there are no other costs involved.

a) If Steve's goal is to have the lowest monthly house payment, which financing offer should Steve take? What is the difference in the monthly cost of the two financing options?

b) If Steve's goal is to save money over the life of the loan, which financing offer should Steve take? What is the difference in the cost of the two financing options over the life of the loan?

Application (Real Life) Assessment Answer

a) Steve will save $394.47 per month by selecting the 30-year loan.

b) Steve will save $42,829 in interest by selecting the 15-year loan.
ALIGNED COURSE EXPECTATION C02

Find the inverse of logarithmic and exponential functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Find the inverse of each of the following functions:

a) \( f(x) = \log_2 x \)

b) \( f(x) = \log_3 \left( 4^x + 4 \right) \)

Computational Assessment Answer

a) \( f^{-1}(x) = \frac{10^x}{2} \)

b) \( f^{-1}(x) = \log_4 \left( 3^x - 4 \right) \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Determine the exponential expression that represents the inverse of each logarithmic expression.

a) \( f(x) = \ln(x - 5) \)

b) \( f(x) = \log(x) - 5 \)

c) \( f(x) = \log_5 x \)

Conceptual Assessment Answer

a) \( f^{-1}(x) = e^x + 5 \)

b) \( f^{-1}(x) = 10^{x-5} \)

c) \( f^{-1}(x) = 5^x \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The number of appetizers, \( a \), that an apprentice appetizer chef can prepare per day after \( t \) days of training is modeled by the equation \( a = 5 \times 6^{3t-4} + 7 \). A student was asked to find the inverse of this equation. His answer was

\[
 t = \frac{5}{6} \ln x - 7.
\]

Is this correct? If not, determine the inverse.

Application (Real Life) Assessment Answer

No. The inverse is:

\[
 t = \frac{\log \left( \frac{x - 7}{5} \right) + 4}{3}.
\]
ALIGNED COURSE EXPECTATION C03

Expand a logarithmic expression into a linear combination of simple logarithms.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Expand each logarithm expression:

a) $\log_4 5x$

b) $\log_{10} \frac{y}{2}$

c) $\log_3 x^2$

Computational Assessment Answer

a) $\log_4 5 + \log_4 x$

b) $\log_{10} y - \log_{10} 2$

c) $2\log_3 x$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Determine which of the following equations are true for every positive base, $b$, and positive values $x$ and $y$.

a) $\log_b (xy) = (\log_b x)(\log_b y)$

b) $\log_b \frac{x}{y} = \log_b x - \log_b y$

c) $y \log_b x = \log_b x^y$
Conceptual Assessment Answer

a) False
b) True
c) True

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The Richter scale is a logarithmic function used to measure the magnitude of earthquakes. The formula for the Richter scale is

\[ R = \log \left( \frac{A}{A_0} \right) \]

where \( A \) is the measure of the amplitude of the earthquake wave and \( A_0 \) is the amplitude of the smallest detectable wave. Rewrite the formula as a linear combination of simple logarithms.

Application (Real Life) Assessment Answer

\[ R = \log A - \log A_0 \]
ALIGNED COURSE EXPECTATION C04

Evaluate a logarithmic expression using properties of logarithms.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Evaluate each logarithmic expression by applying properties of logarithms:

a) $7^\log_7 4$

b) $\ln e^9$

c) $5\log_{10} 1000$

d) $-2\log_3 1$

e) $\log_2 32$

Computational Assessment Answer

a) 4

b) 9

c) 15

d) 0

e) 5

CONCEPTUAL ASSESSMENT

Taxonomy: C

Show that the following two expressions are not equivalent.

$$\frac{\log_4 16}{\log_4 4}$$
Conceptual Assessment Answer

\[ \log_4 \frac{16}{4} = \log_4 4 = 1, \quad \text{however,} \quad \frac{\log_4 16}{\log_4 4} = 2 = 2 \]

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The total cost of maintaining inventory on a specific food item is minimized when the size \( x \) of each order is

\[ \log x = \frac{1}{2} \log 2 + \frac{1}{2} \log C + \frac{1}{2} \log D - \frac{1}{2} \log E. \]

Express \( \log x \) in simplified form.

Application (Real Life) Assessment Answer

\[ \log x = \log \sqrt{\frac{2CD}{E}} \]
ALIGNED COURSE EXPECTATION C05

Evaluate logarithmic equations by rewriting as exponential expressions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the following logarithmic equation: $\log_2 28 - \log_2 7 = 3$.

Computational Assessment Answer

$x = 2$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Only one of the following equations can be solved without a calculator. Select the correct equation and explain how to solve the equation.

a) $x = \log 25$

b) $x = \log_5 25$

Conceptual Assessment Answer

a) This equation cannot be solved without a calculator because $x = \log 25$ has a log with base 10, and 25 cannot be written as a power of 10.

b) This equation can be solved without a calculator since $x = \log_5 25$ has log with base 5, and 25 can be written as a power of 5.

$x = \log_5 25$ can be rewritten in exponential form as $5^x = 25 = 5^2$; therefore, $x = 2$. 
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Jose wants to buy a car worth $7,500. He has $5,000 that he plans to invest so that his money can grow. If the bank’s rate of interest is 4.75 % compounded annually, how many years will it take for the money to grow to $7,500?

Application (Real Life) Assessment Answer

8.74 years
ALIGNED COURSE EXPECTATION C06

Solve logarithmic equations.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve each of the following logarithmic equations. Round answers to the nearest hundredth.

a) \( \log_2(x + 4) = 3 \)

b) \( 2\ln x + \ln 4 = 1 \)

c) \( \log(x - 3) - \log(x + 3) = \log 7 \)

Computational Assessment Answer

a) \( x = 4 \)

b) \( x = \frac{e}{2} \approx 1.36 \)

c) no solution

CONCEPTUAL ASSESSMENT

Taxonomy: C

If \( b \) is any positive real number, then determine the solution to \( x = \log_b 1. \)

Conceptual Assessment Answer

\( x = 0 \)
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The formula

\[ dBA = 10 \log \left( \frac{I}{10^{-12}} \right) \]

can be used to determine the number of decibels, \( dBA \), given the intensity of a sound, \( I \), in watts per square meter. Use this formula to determine the sound intensity of a car horn at 110 decibels.

Application (Real Life) Assessment Answer

\[ I = 10^{23} \text{ watts per square meter} \]
ALIGNED COURSE EXPECTATION C07

Determine the domain and range of a logarithmic function.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Give the domain and range of \( f(x) = \log_2(x-4)+3 \).

Computational Assessment Answer

Domain: \((4, \infty)\)
Range: \((\infty, \infty)\)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Given the function \( f(x) = 3-\ln3x \), determine whether each of the following statements is True or False.

a) \( f(x) \) has a domain of \((3, \infty)\)

b) \( f(x) \) has a range of \((-3, \infty)\)

c) \( f(x) \) has a vertical asymptote at \( x = 0 \)

d) \( f(x) \) has a horizontal asymptote at \( y = 3 \)

Conceptual Assessment Answer

a) False
b) False
c) True
d) False
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

The apparent magnitude, $A$, of a star indicates how bright the star appears when looking at the star from the Earth. The value for the apparent magnitude for a sun with a brightness similar to ours can be calculated by using the following formula, where $d$ represents the distance from the Earth in parsecs:

$$A = 5 \log_{10} d - 0.15.$$ 

If the range of the human eye can detect an apparent magnitude of less than 6, determine the domain and range for this function.

Application (Real Life) Assessment Answer

Domain: $\left[ 0, 10^{1.23} \right) \text{ or } \left[ 0, 16.98 \right)$

Range: $(-\infty, 6)$
RATIONAL EXPRESSIONS & EQUATIONS

• ALIGNED COURSE EXPECTATION D01

Solve rational inequalities.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the inequality \( \frac{3x-5}{x+3} > 2 \).

Computational Assessment Answer

\((-\infty, -3) \cup (11, \infty)\)

CONCEPTUAL ASSESSMENT

Taxonomy: C

Without solving, which solution for the following rational inequality cannot be correct and why?

\( \frac{2x+7}{x-4} \leq 0 \).

a) \([\frac{-7}{2}, 4]\)

b) \([\frac{-7}{2}, 4]\)

Conceptual Assessment Answer

Answer choice (a) includes the value of 4, which would make the rational expression undefined.
APPLICATION ASSESSMENT

Taxonomy: D

When a new cell phone is released, the weekly sales generally increase rapidly for a period of time and then begin to decrease. Suppose that the weekly sales, $W$, (in thousands of units) $x$ weeks after the cell phone is released are given by

$$W = \frac{400x}{x^2 + 300}.$$

When will sales be 10,000 phones per week or more?

**Application (Real Life) Assessment Answer**

From week 10 through week 30.
POLYNOMIAL EXPRESSIONS & EQUATIONS

• ALIGNED COURSE EXPECTATION F01

Identify and solve equations that are quadratic in form, including application problems.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Use the substitution $u= (2x+1)^2$ to solve $(2x+1)^4 - 5(2x+1)^2 = 36$.

Computational Assessment Answer

$x = \frac{-1-2i}{2}, \quad x = \frac{-1+2i}{2}, \quad x = 1, \quad x = -2$

CONCEPTUAL ASSESSMENT

Taxonomy: C

Find $g(x)$ so that the substitution $u = g(x)$ could be used to rewrite $2e^{2x} - 5e^x + 1 = 0$ as a quadratic equation in $u$.

Conceptual Assessment Answer

$g(x) = e^x$

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

Suppose $ax^2 + bx + c = 0$ has two distinct negative solutions. What can be said about the solutions to $ax^4 + bx^2 + c = 0$?

Application (Real Life) Assessment Answer

The equation $ax^4 + bx^2 + c = 0$ will have four complex roots.
ALIGNED COURSE EXPECTATION F02

Apply Rational Root Theorem, Factor Theorem, and synthetic division to find all roots (zeros) of polynomial functions.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Consider the polynomial function \( f(x) = x^3 + 3x^2 + 9x - 13. \)

a) Use the Rational Root Theorem to list all possible rational zeros of the polynomial function.

b) Use synthetic division to identify a rational zero of the polynomial function.

c) Use the results above to find all remaining zeros of the polynomial function.

Computational Assessment Answer

a) The possible rational zeros are \( \{-13, -1, 1, 13\}. \)

b) \( 1 \)

c) The zeros are \( 1, -2-3i, -2+3i. \)

CONCEPTUAL ASSESSMENT

Taxonomy: C

The function \( f(x) = x^3 + 3x^2 + 9x - 13 \) has at least one complex root. How many more distinct roots does it have, and what type are they?

Conceptual Assessment Answer

This function has two more distinct roots: one complex and one real.
APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

A rectangular prism has a volume of 32 cubic inches. If the length is 3 inches more than the width, and the height is 4 inches more than the length, find the dimensions of the prism.

Application (Real Life) Assessment Answer

The width is 1 inch; the length is 4 inches; and the height is 8 inches.
**ALIGNED COURSE EXPECTATION F03**

Factor a polynomial using the Factor Theorem.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Use the Factor Theorem to completely factor the polynomial expression $3x^3 - 19x^2 + 30x - 8$.

**Computational Assessment Answer**

Factored form: $(3x - 1)(x - 2)(x - 4)$

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

Given that $(x - c)$ is a factor of the polynomial $p(x)$, what is the remainder of $p(x) ÷ (x - c)$?

**Conceptual Assessment Answer**

0

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: B**

Given a cubic function, $f(x)$, that has roots of $-2$ and $2 + i$, and for which $f(0) = 6$, find the function.

**Application (Real Life) Assessment Answer**

$$f(x) = \frac{3}{5}x^3 - \frac{6}{5}x^2 - \frac{9}{5}x + 6$$
**ALIGNED COURSE EXPECTATION F04**

Evaluate a polynomial function at a specified number by using synthetic division and the Remainder Theorem.

**COMPUTATIONAL ASSESSMENT**

**Taxonomy: A**

Use synthetic division and the Remainder Theorem to evaluate the polynomial $f(x) = 4x^4 - 2x^3 + x^2 + 4$ at $x = -1$.

**Computational Assessment Answer**

11

**CONCEPTUAL ASSESSMENT**

**Taxonomy: C**

If $(x - c)$ is a factor of the polynomial $p(x)$, what is $p(c)$?

**Conceptual Assessment Answer**

0

**APPLICATION (REAL LIFE) ASSESSMENT**

**Taxonomy: B**

Let $f(x) = x^3 - 4x + c$. Use synthetic division to find the value of $c$ for which $f(4) = 70$.

**Application (Real Life) Assessment Answer**

22
**ALIGNED COURSE EXPECTATION F05**

Graph a polynomial function.

**COMPUTATIONAL ASSESSMENT**

Taxonomy: A

Sketch a graph of \( f(x) = -\frac{1}{4}x^5 + x^4 + \frac{3}{4}x^3 - \frac{9}{2}x^2 \).

Computational Assessment Answer

![Graph of the polynomial function](image-url)
CONCEPTUAL ASSESSMENT

Taxonomy: C

Given the graph of \( f(x) \) below, determine the effect on each of the following when multiplying the entire function by \( x \).

a) Number of \( x \)-intercepts

b) Number of turns

c) End behavior

Conceptual Assessment Answer

a) The number of \( x \)-intercepts would stay the same.

b) The number of turns would increase by two.

c) The end behavior would change to: As \( x \to -\infty, f(x) \to \infty \); and as \( x \to \infty, f(x) \to \infty \).

APPLICATION (REAL LIFE) ASSESSMENT

No assessment.
ALIGNED COURSE EXPECTATION F06

Solve polynomial inequalities of degree two or higher.

COMPUTATIONAL ASSESSMENT

Taxonomy: A

Solve the polynomial inequality \( x^3 - x^2 - 4x \leq -4 \).

Computational Assessment Answer

The solution set is \( \{ x | 1 \leq x \leq 2 \text{ or } x \leq -2 \} \) or \( (-\infty, -2] \cup [1, 2] \).

CONCEPTUAL ASSESSMENT

Taxonomy: C

Consider the graph of \( f(x) \) below:

Use the graph to solve \( f(x) \leq 0 \).
Conceptual Assessment Answer

The solution set is $(-\infty, -2] \cup [1, 2]$.

APPLICATION (REAL LIFE) ASSESSMENT

Taxonomy: D

For safety purposes, fireworks are timed to explode above a height of 40 meters. Suppose a firework is fired straight up from the ground. Its height is given by the formula $h(t) = -4.9t^2 + 98t$. To the nearest tenth of a second, for what range of times can the firework be safely detonated?

Application (Real Life) Assessment Answer

The firework may be safely detonated between 0.4 seconds and 19.6 seconds after launch.
APPENDICES

Appendix A: Multicourse Alignment Tables................................................................. 320
Appendix B: Aligned Course Expectations by Category............................................. 340
Appendix C: Mathematics Professional Alignment Council...................................... 349
Teams explored the vertical alignment of these gateway courses by comparing exit-level Algebra II Aligned Course Expectations (ACEs) to entry-level College Algebra and Precalculus ACEs. To achieve this, all partnerships gathered at the beginning of the 2014 academic year to identify major topics in each of the ACE categories that persist from Algebra II through College Algebra and into Precalculus. For example, “solving polynomial equations by factoring” is a skill that first appears in Algebra II in the form “solving quadratic equations by factoring” and eventually evolves into “solving higher degree polynomial equations using synthetic division and the Factor Theorem.” While these skills are quite different, they share a common thread, which we have termed a “topic.”

Individual mathematics professional alignment councils (MPACs) met throughout the first half of the 2014 academic year to construct multicourse alignment tables for each topic in each category. As with ACEs, partnerships reconvened toward the end of this process to analyze and revise these tables. The result is a collection of tables demonstrating the alignment of skills through the evolution of topics.

**CATEGORY A: FUNCTIONS & MISCELLANY**

**Topic: Create representations of functions.**

<table>
<thead>
<tr>
<th>Topic Description</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A01—Transform between representations of functions (including table of values, equation, inequality, graph, verbal description).</td>
<td>A05—Transform between representations of functions, e.g., table of values, equation, graph, verbal description.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>

**Topic: Evaluate functions.**

<table>
<thead>
<tr>
<th>Topic Description</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A02—Evaluate functions at a given input value.</td>
<td>A06—Evaluate functions given an input value.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>
### Topic: Understand relationship between zeros and $x$-intercepts.

<table>
<thead>
<tr>
<th>Course</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A09—Transform between zeros and $x$-intercepts of functions.</td>
<td>A07—Transform between zeros and $x$-intercepts of functions.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>

### Topic: Evaluate and graph piecewise functions.

<table>
<thead>
<tr>
<th>Course</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>A02—Evaluate a given piecewise defined function.</td>
<td>A08—Evaluate and graph a given piecewise function.</td>
<td></td>
</tr>
<tr>
<td>A08—Evaluate and graph a given piecewise defined function.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Topic: Determine intervals of increase/decrease.

<table>
<thead>
<tr>
<th>Course</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>A03—From a graph, determine the intervals on which a given function is increasing, decreasing, and constant.</td>
<td>No corresponding ACE at this level</td>
<td></td>
</tr>
</tbody>
</table>
### Topic: Find the composition of functions.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>A05—Find the composition of two functions.</td>
<td>A04—Find the composition of two functions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A06—Decompose a given function into a composition of simpler functions.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Topic: Evaluate and graph parent functions and transformations.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A06—Identify the type of parent function presented given unique characteristics of that parent function.</td>
<td>A10—Discriminate between constant, linear, and quadratic parent functions and their graphs.</td>
<td>A01—Identify the parent function (square, cube, square root, cube root, absolute value, reciprocal, exponential/logarithmic) and transformations (translations, reflections, dilations) for a given transformed function and use them to graph the functions.</td>
<td>A05—Identify the parent function (square, cube, square root, cube root, absolute value, reciprocal, exponential/logarithmic) and transformations (translations, reflections, dilations) for a given transformed function and use them to graph the functions.</td>
<td></td>
</tr>
<tr>
<td>A07—Graph parent functions, including square, absolute value, square root, cube root, cubic, reciprocal, logarithm, and exponential functions.</td>
<td>A08—Graph various transformations of parent functions, including linear, quadratic, exponential, square root, absolute value, and logarithmic functions.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A08—Graph various transformations of parent functions, including linear, quadratic, exponential, square root, absolute value, and logarithmic functions.</td>
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</tr>
</tbody>
</table>
### Topic: Identify domain and range.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A03—Identify the domain and range of a function from various representations, including: table of values, equation, inequality, graph, verbal description.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A05—Identify the reasonable domain and range for a situation modeled by a function, including continuous and discrete situations.</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>A08—Given a table, mapping diagram, or graph, determine the domain and range.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>A07—Given a relation, determine the domain and range.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A03—Determine the domain and range of a relation given a table, graph, or equation.</td>
<td></td>
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</tr>
</tbody>
</table>

### Topic: Apply functions to model real-world situations.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11—Use symbols to represent unknowns and variables in the context of real world applications.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A12—Describe functional relationships for real world situations by writing equations or inequalities.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A04—Identify an appropriate function to fit given data.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A01—Use symbols to represent unknowns and variables in the context of real world applications.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A09—Describe functional relationships for given real world situations and write equations or inequalities; then sketch graphs to answer questions arising from the situation.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>No corresponding ACE at this level.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A06—Apply polynomial, logarithmic, and exponential functions to model real life data.</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
### Topic: Simplify algebraic expressions.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A13—Simplify and perform operations on rational expressions.</td>
<td>A03—Simplify algebraic expressions.</td>
<td>A08—Simplify algebraic expressions.</td>
<td>A02—Simplify algebraic expressions.</td>
<td></td>
</tr>
<tr>
<td>A10—Simplify an algebraic expression using the rules of exponents.</td>
<td>A02—Perform arithmetic operations on integers and rational numbers without using a calculator.</td>
<td></td>
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</tr>
</tbody>
</table>

### Topic: Determine symmetry.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>A04—Determine whether a given function is even, odd, or neither.</td>
<td>A09—Determine the symmetry of a relation given the graph or equation.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A09—Determine symmetry of a relation given the graph or equation.</td>
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<td></td>
</tr>
</tbody>
</table>

### Topic: Perform complex arithmetic.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>A11—Perform basic operations involving complex numbers.</td>
<td>A01—Perform basic operations involving complex numbers.</td>
<td></td>
</tr>
</tbody>
</table>
### CATEGORY B: SYSTEMS

#### Topic: Solve systems of equations in two or three variables.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>B01—Solve a system of two equations in two variables using graphs.</td>
<td>A04—Use arithmetic operations to solve and manipulate equations of two variables.</td>
<td>B01—Solve a system of linear equations using Gauss-Jordan Elimination.</td>
<td>B01—Solve a system of linear equations.</td>
</tr>
<tr>
<td>B02—Solve a system of two equations in two variables using tables.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B03—Solve a system of two equations in two variables using substitution and elimination.</td>
<td>B02—Solve a system of two equations in two variables using substitution or elimination.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B04—Solve a system of two equations in two variables using matrices.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B07—Solve a system of three linear equations in three variables.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Topic: Use systems to model real-world situations.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>B05—Create a system of two equations in two variables to represent a problem situation.</td>
<td>B01—Convert verbal real world situations to linear systems of equations.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
</tr>
<tr>
<td>B06—Solve a real world application problem involving a linear system.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Topic: Recognize the three systems of linear equations.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>B08—Distinguish between inconsistent, dependent, and independent systems of linear equations.</td>
<td>No corresponding ACE at this level.</td>
<td>B02—Distinguish between inconsistent, dependent, and independent systems of linear equations.</td>
<td>B03—Describe the (infinitely many) solutions to dependent systems of linear equations.</td>
<td>No corresponding ACE at this level.</td>
</tr>
</tbody>
</table>

### CATEGORY C: EXPONENTIAL & LOGARITHMIC EXPRESSIONS & EQUATIONS

### Topic: Solve exponential equations, including in applications.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C03—Solve exponential equations using algebraic methods.</td>
<td>No corresponding ACE at this level.</td>
<td>C03—Solve exponential equations by applying the One-To-One Property.</td>
<td>C04—Solve exponential equations using logarithms, including applications.</td>
<td>C01—Solve an exponential equation (including in real world situations).</td>
</tr>
<tr>
<td>C04—Write an exponential model given key characteristics, i.e., base, rate of growth/decay, data points, etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C07—Transform an exponential equation into logarithmic form and vice versa.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Topic: Expand/condense logarithmic expressions.**

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>C06—Condense a linear combination of simple logarithms into a single logarithm.</td>
<td>C03—Expand a logarithmic expression into a linear combination of simple logarithms.</td>
</tr>
</tbody>
</table>

**Topic: Solve logarithmic equations, including in applications.**

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C07—Transform an exponential equation into logarithmic form and vice versa.</td>
<td>No corresponding ACE at this level.</td>
<td>C07—Solve logarithmic equations by rewriting in exponential form, including applications.</td>
<td>C06—Solve logarithmic equations.</td>
</tr>
<tr>
<td>C09—Identify a reasonable domain and range for a situation modeled by a logarithmic function.</td>
<td></td>
<td>C09—Solve logarithmic equations by applying the One-To-One Property.</td>
<td></td>
</tr>
</tbody>
</table>

**Topic: Evaluate logarithmic expressions.**

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01—Evaluate logarithmic expressions by applying properties of logarithms.</td>
<td>No corresponding ACE at this level.</td>
<td>C05—Evaluate logarithmic expressions by applying the Change of Base Formula.</td>
<td>C04—Evaluate a logarithmic expression using properties of logarithms.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C10—Evaluate logarithmic expressions by applying properties of logarithms.</td>
<td>C05—Evaluate logarithmic equations by rewriting as exponential expressions.</td>
</tr>
</tbody>
</table>
**Topic: Find the inverse of an exponential/logarithmic function using algebraic methods.**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C05—Determine the inverse relationship between exponential and logarithmic equations using algebraic methods, graphing, and tables.</td>
<td>No corresponding ACE at this level.</td>
<td>C01—Find inverses of logarithmic and exponential functions.</td>
<td>C02—Find the inverse of logarithmic and exponential functions.</td>
<td></td>
</tr>
<tr>
<td>C06—Transform an exponential equation into logarithmic form and vice versa.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Topic: Simplify exponential expressions.**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C08—Simplify exponential expressions by applying laws of exponents.</td>
<td>C01—Simplify exponential expressions by applying laws of exponents.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>

**Topic: Identify domain and range of an exponential and logarithmic function.**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C02—Identify the domain and range for an exponential equation.</td>
<td>No corresponding ACE at this level.</td>
<td>C02—Find domain and range of exponential and logarithmic functions.</td>
<td>C06—Determine the domain and range of a logarithmic function.</td>
<td></td>
</tr>
<tr>
<td>C05—Identify a reasonable domain and range for a real world situation modeled by an exponential function.</td>
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</tr>
</tbody>
</table>
## CATEGORY D: RATIONAL EXPRESSIONS & EQUATIONS

### Topic: Graph a rational function.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>D04—Graph a rational function from its equation in the form ( f(x) = \frac{1}{x} ) or ( f(x) = \frac{1}{x^2} ), including functions with transformations.</td>
<td>No corresponding ACE at this level.</td>
<td>D01—Graph a rational function by finding or creating the following: asymptotes, end and local behaviors, removable discontinuity, and key points.</td>
<td>No corresponding ACE at this level.</td>
</tr>
</tbody>
</table>

### Topic: Describe the attributes of a rational function.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>D01—Identify the attributes of a given rational function (asymptotes, intercepts, discontinuities, end behavior) from its graph and/or equation.</td>
<td>No corresponding ACE at this level.</td>
<td>D02—Find vertical and horizontal asymptotes of rational functions.</td>
<td>No corresponding ACE at this level.</td>
</tr>
<tr>
<td>D06—Identify the domain and range for a rational function in interval notation and inequalities.</td>
<td></td>
<td>D03—Determine the domain and range of rational functions.</td>
<td></td>
</tr>
</tbody>
</table>
**Topic: Write the equation of a rational function.**

<table>
<thead>
<tr>
<th>Topic: Write the equation of a rational function.</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>D02—Determine the equation of a rational function of the form $f(x) = \frac{1}{x}$ or $f(x) = \frac{1}{x^2}$, including functions with transformations.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>

**Topic: Solve rational equations.**

<table>
<thead>
<tr>
<th>Topic: Solve rational equations.</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>D09—Solve rational equations using algebraic methods.</td>
<td>No corresponding ACE at this level.</td>
<td>D04—Solve rational equations using algebraic, graphic, and tabular methods.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
<tr>
<td>D05—Evaluate rational functions given specific inputs/outputs.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Topic: Solve rational inequalities.**

<table>
<thead>
<tr>
<th>Topic: Solve rational inequalities.</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>D08—Solve rational inequalities.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>D01—Solve rational inequalities.</td>
<td></td>
</tr>
</tbody>
</table>
### Topic: Solve applications with rational functions.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>D10—Use inverse variations (i.e., $k=xy$) to solve application problems, such as Boyle’s Law.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
<tr>
<td>D07—Identify a reasonable domain and range for a real world situation modeled by a rational function.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### CATEGORY E: RADICAL EXPRESSIONS & EQUATIONS

#### Topic: Identify domain and range.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>E04—Identify the domain and range for a square root function and express in interval notation and in terms of inequalities.</td>
<td>No corresponding ACE at this level.</td>
<td>E02—Find the domain and range of radical functions.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>

#### Topic: Represent functions.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>E02—Create a representation of a square root function (table of values, equation, graph, verbal description) given the same function in a different format.</td>
<td>No corresponding ACE at this level.</td>
<td>E01—Create multiple representations (table, equation, graph, verbal description) of a radical function given the same function in a different format.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
</tbody>
</table>
### Topic: Identify inverse functions.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>E01—Find the inverse of a square root function using algebraic methods.</td>
<td>No corresponding ACE at this level.</td>
<td>E03—Find inverses of radical functions.</td>
<td>A07—Algebraically determine the inverse of a function.</td>
</tr>
</tbody>
</table>

### Topic: Simplify radical expressions.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>E01—Simplify radical expressions.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
</tr>
</tbody>
</table>

### Topic: Solve radical equations.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>E03—Solve square root equations and inequalities using algebraic methods, and recognize extraneous solutions.</td>
<td>E02—Solve square root equations.</td>
<td>E04—Solve radical equations.</td>
<td>No corresponding ACE at this level.</td>
</tr>
</tbody>
</table>
## CATEGORY F: POLYNOMIAL EXPRESSIONS & EQUATIONS

### Topic: Find roots of polynomial functions by factoring.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F08—Solve quadratic equations by factoring.</td>
<td>F02—Find the greatest common factor of a polynomial expression.</td>
<td>F03—Apply the Rational Root Theorem, Factor Theorem, and synthetic division to find all roots (zeros) of polynomial functions.</td>
<td>F01—Identify and solve equations that are quadratic in form, including application problems.</td>
</tr>
<tr>
<td></td>
<td>F09—Solve quadratic equations by factoring.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Topic: Solve quadratic equations using completing the square.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F07—Solve a quadratic equation by applying the square root property.</td>
<td>No corresponding ACE at this level.</td>
<td>F08—Solve a quadratic equation by completing the square.</td>
<td>F01—Identify and solve equations that are quadratic in form, including application problems.</td>
</tr>
</tbody>
</table>
### Topic: Use Quadratic Formula.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F10—Solve quadratic equations and applications (projectile motion, geometric area, Pythagorean Theorem, etc.) using the Quadratic Formula.</td>
<td>No corresponding ACE at this level.</td>
<td>F09—Solve quadratic equations and applications (projectile motion, geometric area, Pythagorean Theorem, etc.) using the algebraic method of the Quadratic Formula.</td>
<td>F01—Identify and solve equations that are quadratic in form, including application problems.</td>
</tr>
</tbody>
</table>

F13—Use the discriminant to describe the types of roots of a quadratic function.

### Topic: Solve polynomial inequalities.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F12—Solve quadratic inequalities.</td>
<td>No corresponding ACE at this level.</td>
<td>F07—Solve a polynomial inequality of degree two or higher.</td>
<td>F06—Solve polynomial inequalities of degree two or higher.</td>
</tr>
</tbody>
</table>

### Topic: Find the domain and range of quadratic functions.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F03—Find the domain and range of quadratic functions.</td>
<td>F08—Given the graph of a quadratic function, find the domain and range.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
</tr>
</tbody>
</table>

### Topic: Evaluate polynomial functions.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>F01—Evaluate polynomial functions by substitution or by synthetic division and the Remainder Theorem.</td>
<td>F04—Evaluate a polynomial function at a specified number by using synthetic division and the Remainder Theorem.</td>
</tr>
</tbody>
</table>
## Topic: Find the inverse of a quadratic function.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F14—Find the inverse of a quadratic function using algebraic methods.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
<tr>
<td>F15—Verify whether a given quadratic function with a restricted domain and a square root function are inverses of each other.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

## Topic: Identify $x$- and $y$-intercepts of polynomial functions.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Algebra II Exit ACE</th>
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<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F11—Find $x$- and $y$-intercepts of a quadratic function.</td>
<td>F05—Determine the $x$- and $y$-intercepts of a linear function.</td>
<td>F04—Identify the $x$-intercepts (if applicable) and $y$-intercept of polynomial functions.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
<tr>
<td>F06—Given the graph of a quadratic equation, students can find the $y$-intercept.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>F07—Given the graph of a quadratic equation, students can find the $x$-intercept(s).</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
### Topic: Model application problems involving quadratic equations.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
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<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F05—Create a model of a real world situation by applying quadratic functions.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
<tr>
<td>F06—Identify the domain and range for a real world situation modeled by a quadratic function.</td>
<td></td>
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</tr>
</tbody>
</table>

### Topic: Write polynomial functions given characteristics.

<table>
<thead>
<tr>
<th></th>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F01—Write a quadratic function given key characteristics, including vertex, direction, points on the parabola, or roots.</td>
<td>No corresponding ACE at this level.</td>
<td>F05—Determine the equation of a polynomial function from given roots (real or complex), including in an application.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
</tr>
<tr>
<td>F02—Describe a quadratic function using multiple representations, such as a table of values, an equation, a graph, or a verbal description.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
### Topic: Graph polynomial equations.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
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<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F09—Graph quadratic functions.</td>
<td>F03—Graph linear equations.</td>
<td>F04—Identify the $x$-intercepts (if applicable) and $y$-intercept of polynomial functions.</td>
<td>F05—Graph a polynomial function.</td>
</tr>
<tr>
<td></td>
<td>F04—Write a linear equation given the graph of a line.</td>
<td>F06—Describe end behavior of a polynomial function.</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>F10—State the maximum number of turning points for a given polynomial function.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>F11—Use the multiplicity of a zero to determine the behavior of the graph of a polynomial function at each $x$-intercept.</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>F12—Graph a polynomial function.</td>
<td></td>
</tr>
</tbody>
</table>

### Topic: Transform between various forms of polynomial functions.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>F02—Describe a quadratic function using multiple representations, such as a table of values, an equation, a graph, or a verbal description.</td>
<td>F01—Write polynomial expressions in standard format: $ax^n + bx^{n-1} + cx^{n-2} + ...$</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
</tr>
<tr>
<td>F04—Transform one form of a quadratic equation (i.e., vertex form, standard form) into the other form.</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
### Topic: Find roots of higher degree polynomials.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>F02—Factor a polynomial of degree three or higher in using the Factor Theorem.</td>
<td>F02—Apply the Rational Root Theorem, the Factor Theorem, and synthetic division to find all roots (zeros) of polynomial functions.</td>
</tr>
</tbody>
</table>

### CATEGORY G: CONIC SECTIONS

### Topic: Transform conic equations into various forms.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>G01—Use completing the square to transform a conic equation in general form into an equation in standard form and vice versa.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
<td>No corresponding ACE at this level.</td>
</tr>
</tbody>
</table>

### Topic: Graph conic sections.

<table>
<thead>
<tr>
<th>Algebra II Exit ACE</th>
<th>College Algebra Entrance ACE</th>
<th>College Algebra Exit ACE</th>
<th>Precalculus Entrance ACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>G02—Graph a circle given an equation in standard or general form (no rotations).</td>
<td>No corresponding ACE at this level.</td>
<td>G01—Given the equation of a circle, identify the center and radius of the circle and sketch the graph.</td>
<td>No corresponding ACE at this level.</td>
</tr>
<tr>
<td>G03—Graph a parabola given an equation in standard or general form.</td>
<td>No corresponding ACE at this level.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Topic: Write conics in standard form.</td>
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<tr>
<td>--------------------------------------</td>
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<td></td>
</tr>
<tr>
<td>Algebra II Exit ACE</td>
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<tr>
<td>College Algebra Entrance ACE</td>
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<td></td>
</tr>
<tr>
<td>College Algebra Exit ACE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precalculus Entrance ACE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G04—Write the equation of a circle in standard form given its attributes (center, radius, or graph).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No corresponding ACE at this level.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G05—Write the equation of a parabola in standard form given its attributes (vertex, focus, direction, latus rectum, endpoints, etc.).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G02—Determine the equation of the circle given geometric information.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No corresponding ACE at this level.</td>
<td></td>
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</tr>
</tbody>
</table>
# Aligned Course Expectations by Category

## Exit-Level Algebra II Aces

### Category A: Functions & Miscellany

<table>
<thead>
<tr>
<th>A01</th>
<th>Transform between representations of functions (including table of values, equation, inequality, graph, verbal description).</th>
</tr>
</thead>
<tbody>
<tr>
<td>A02</td>
<td>Evaluate functions at a given input value.</td>
</tr>
<tr>
<td>A03</td>
<td>Identify the domain and range of a function from various representations, including table of values, equation, inequality, graph, verbal description.</td>
</tr>
<tr>
<td>A04</td>
<td>Identify an appropriate function to fit given data.</td>
</tr>
<tr>
<td>A05</td>
<td>Identify the reasonable domain and range for a situation modeled by a function, including continuous and discrete situations.</td>
</tr>
<tr>
<td>A06</td>
<td>Identify the type of parent function presented given unique characteristics of that parent function.</td>
</tr>
<tr>
<td>A07</td>
<td>Graph parent functions, including square, absolute value, square root, cube root, cubic, reciprocal, logarithm, and exponential functions.</td>
</tr>
<tr>
<td>A08</td>
<td>Graph various transformations of parent functions, including linear, quadratic, exponential, square root, absolute value, and logarithmic functions.</td>
</tr>
<tr>
<td>A09</td>
<td>Transform between zeros and $x$-intercepts of functions.</td>
</tr>
<tr>
<td>A10</td>
<td>Simplify an algebraic expression using the rules of exponents.</td>
</tr>
<tr>
<td>A11</td>
<td>Use symbols to represent unknowns and variables in the context of real world applications.</td>
</tr>
<tr>
<td>A12</td>
<td>Describe functional relationships for real world situations by writing equations or inequalities.</td>
</tr>
<tr>
<td>A13</td>
<td>Simplify and perform operations on rational expressions.</td>
</tr>
</tbody>
</table>

### Category B: Systems

<table>
<thead>
<tr>
<th>B01</th>
<th>Solve a system of two equations in two variables using graphs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B02</td>
<td>Solve a system of two equations in two variables using tables.</td>
</tr>
<tr>
<td>B03</td>
<td>Solve a system of two equations in two variables using substitution and elimination.</td>
</tr>
<tr>
<td>B04</td>
<td>Solve a system of two equations in two variables using matrices.</td>
</tr>
<tr>
<td>B05</td>
<td>Create a system of two equations in two variables to represent a problem situation.</td>
</tr>
</tbody>
</table>
### APPENDIX B • ACES BY CATEGORY

#### Category B: Systems

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B06</td>
<td>Solve a real world application problem involving a linear system.</td>
</tr>
<tr>
<td>B07</td>
<td>Solve a system of three linear equations in three variables.</td>
</tr>
<tr>
<td>B08</td>
<td>Distinguish between inconsistent, dependent, and independent systems of linear equations.</td>
</tr>
</tbody>
</table>

#### Category C: Exponential & Logarithmic Expressions & Equations

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>Evaluate logarithmic expressions by applying properties of logarithms.</td>
</tr>
<tr>
<td>C02</td>
<td>Identify the domain and range for an exponential equation.</td>
</tr>
<tr>
<td>C03</td>
<td>Solve exponential equations using algebraic methods.</td>
</tr>
<tr>
<td>C04</td>
<td>Write an exponential model given key characteristics, i.e., base, rate of growth/decay, data points, etc.</td>
</tr>
<tr>
<td>C05</td>
<td>Identify a reasonable domain and range for a real world situation modeled by an exponential function.</td>
</tr>
<tr>
<td>C06</td>
<td>Determine the inverse relationship between exponential and logarithmic equations using algebraic methods, graphing, and tables.</td>
</tr>
<tr>
<td>C07</td>
<td>Transform an exponential equation into logarithmic form and vice versa.</td>
</tr>
<tr>
<td>C08</td>
<td>Simplify exponential expressions by applying laws of exponents.</td>
</tr>
<tr>
<td>C09</td>
<td>Identify a reasonable domain and range for a situation modeled by a logarithmic function.</td>
</tr>
</tbody>
</table>

#### Category D: Rational Expressions & Equations

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D01</td>
<td>Identify the attributes of a given rational function (asymptotes, intercepts, discontinuities, end behavior) from its graph and/or equation.</td>
</tr>
<tr>
<td>D02</td>
<td>Determine the equation of a rational function in the form ( f(x) = \frac{1}{x} ) or ( f(x) = \frac{1}{x} ) from its graph using transformations.</td>
</tr>
<tr>
<td>D03</td>
<td>Convert a rational function from a ratio of two polynomials to a polynomial plus remainder and vice versa.</td>
</tr>
<tr>
<td>D04</td>
<td>Graph a rational function from its equation in the form ( f(x) = \frac{1}{x} ) or ( f(x) = \frac{1}{x} ), including functions with transformations.</td>
</tr>
<tr>
<td>D05</td>
<td>Evaluate rational functions given specific inputs/outputs.</td>
</tr>
<tr>
<td>D06</td>
<td>Identify the domain and range for a rational function in interval notation and inequalities.</td>
</tr>
<tr>
<td>D07</td>
<td>Identify a reasonable domain and range for a real world situation modeled by a rational function.</td>
</tr>
<tr>
<td>D08</td>
<td>Solve rational inequalities.</td>
</tr>
</tbody>
</table>
### Category D: Rational Expressions & Equations

<table>
<thead>
<tr>
<th>D09</th>
<th>Solve rational equations using algebraic methods.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D10</td>
<td>Use inverse variations (i.e., $k=xy$) to solve application problems, such as Boyle’s Law.</td>
</tr>
</tbody>
</table>

### Category E: Radical Expressions & Equations

<table>
<thead>
<tr>
<th>E01</th>
<th>Find the inverse of a square root function using algebraic methods.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E02</td>
<td>Create a representation of a square root function (table of values, equation, graph, verbal description) given the same function in a different format.</td>
</tr>
<tr>
<td>E03</td>
<td>Solve square root equations and inequalities using algebraic methods, and recognize extraneous solutions.</td>
</tr>
<tr>
<td>E04</td>
<td>Identify the domain and range for a square root function and express in interval notation and in terms of inequalities.</td>
</tr>
</tbody>
</table>

### Category F: Polynomial Expressions & Equations

<table>
<thead>
<tr>
<th>F01</th>
<th>Write a quadratic function given key characteristics, i.e., vertex, direction, points on the parabola, or roots.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F02</td>
<td>Describe a quadratic function using multiple representations, such as a table of values, an equation, a graph, or a verbal description.</td>
</tr>
<tr>
<td>F03</td>
<td>Find the domain and range of quadratic functions.</td>
</tr>
<tr>
<td>F04</td>
<td>Transform one form of a quadratic equation, i.e., vertex form and standard form, into the other form.</td>
</tr>
<tr>
<td>F05</td>
<td>Create a model of a real world situation by applying quadratic functions.</td>
</tr>
<tr>
<td>F06</td>
<td>Identify the domain and range for a real world situation modeled by a quadratic function.</td>
</tr>
<tr>
<td>F07</td>
<td>Solve a quadratic equation by applying the square root property.</td>
</tr>
<tr>
<td>F08</td>
<td>Solve quadratic equations by factoring.</td>
</tr>
<tr>
<td>F09</td>
<td>Graph quadratic functions.</td>
</tr>
<tr>
<td>F10</td>
<td>Solve quadratic equations and applications (projectile motion, geometric area, Pythagorean Theorem, etc.) using the Quadratic Formula.</td>
</tr>
<tr>
<td>F11</td>
<td>Find $x$- and $y$-intercepts of a quadratic function.</td>
</tr>
<tr>
<td>F12</td>
<td>Solve quadratic inequalities.</td>
</tr>
<tr>
<td>F13</td>
<td>Use the discriminant to describe the types of roots of a quadratic function.</td>
</tr>
</tbody>
</table>
Category F: Polynomial Expressions & Equations

| F14 | Find the inverse of a quadratic function using algebraic methods. |
| F15 | Verify whether a given quadratic function with a restricted domain and a square root function are inverses of each other. |

Category G: Conic Sections

| G01 | Use completing the square to transform a conic equation in general form into an equation in standard form and vice versa. |
| G02 | Graph a circle given an equation in standard or general form (no rotations). |
| G03 | Graph a parabola given an equation in standard or general form. |
| G04 | Write the equation of a circle in standard form given its attributes (center, radius, or graph). |
| G05 | Write the equation of a parabola in standard form given its attributes (vertex, focus, direction, latus rectum, endpoints, etc.). |

ENTRY-LEVEL COLLEGE ALGEBRA ACES

Category A: Functions & Miscellany

| A01 | Use symbols to represent unknowns and variables in the context of real world applications. |
| A02 | Perform arithmetic operations on integers and rational numbers without using a calculator. |
| A03 | Simplify algebraic expressions. |
| A04 | Use arithmetic operations to solve and manipulate equations of two variables. |
| A05 | Transform between representations of functions, e.g., table of values, equation, graph, verbal description. |
| A06 | Evaluate functions given an input value. |
| A07 | Transform between zeros and x-intercepts of functions. |
| A08 | Given a table, mapping diagram, or graph, determine the domain and range. |
| A09 | Describe functional relationships for given problem situations, and write equations or inequalities to which could be used to solve real world problems. |
| A10 | Discriminate between constant, linear, and quadratic parent functions and their graphs. |
### Category B: Systems

| B01 | Convert verbal real world situations to linear systems of equations. |

### Category C: Exponential & Logarithmic Expressions & Equations

| C01 | Simplify exponential expressions by applying laws of exponents. |

### Category D: Rational Expressions & Equations

| No ACEs in Category D |

### Category E: Radical Expressions & Equations

| E01 | Simplify radical expressions. |
| E02 | Solve square root equations. |

### Category F: Polynomial Expressions & Equations

| F01 | Write polynomial expressions in standard format, \( ax^n + bx^{n-1} + cx^{n-2} + \ldots \) |
| F02 | Find the greatest common factor of a polynomial expression. |
| F03 | Graph linear equations. |
| F04 | Write a linear equation given the graph of a line. |
| F05 | Determine the \(x\)- and \(y\)-intercepts of a linear function. |
| F06 | Given the graph of a quadratic equation, students can find the \(y\)-intercept. |
| F07 | Given the graph of a quadratic equation, students can find the \(x\)-intercept(s). |
| F08 | Given the graph of a quadratic function, find the domain and range. |
| F09 | Solve quadratic equations by factoring. |

### Category G: Conic Sections

| No ACEs in Category G |
## APPENDIX B • ACES BY CATEGORY

### EXIT-LEVEL COLLEGE ALGEBRA ACES

#### Category A: Functions & Miscellany

| A01 | Identify the parent function (square, cube, square root, cube root, absolute value, reciprocal, exponential/logarithmic) and transformations (translations, reflections, dilations) for a given transformed function and use them to graph the functions. |
| A02 | Evaluate a given piecewise defined function. |
| A03 | From a graph, determine the intervals on which a given function is increasing, decreasing, and constant. |
| A04 | Determine whether a given function is even, odd, or neither. |
| A05 | Find the composition of two functions. |
| A06 | Decompose a given function into a composition of simpler functions. |
| A07 | Given a relation, determine the domain and range. |
| A08 | Simplify algebraic expressions. |
| A09 | Determine symmetry of a relation given the graph or equation. |
| A10 | Graph a given piecewise defined function. |
| A11 | Perform basic operations involving complex numbers. |

#### Category B: Systems

| B01 | Solve a system of linear equations using Gauss-Jordan Elimination. |
| B02 | Distinguish between inconsistent, dependent, and independent systems of linear equations. |
| B03 | Describe the (infinitely many) solutions to dependent systems of linear equations. |

#### Category C: Exponential & Logarithmic Expressions & Equations

| C01 | Find inverses of logarithmic and exponential functions. |
| C02 | Find domain and range of exponential and logarithmic functions. |
| C03 | Solve exponential equations by applying the One-To-One Property. |
| C04 | Solve exponential equations using logarithms, including applications. |
| C05 | Evaluate logarithmic expressions by applying the Change of Base Formula. |
| C06 | Condense a linear combination of simple logarithms into a single logarithm. |
### Category C: Exponential & Logarithmic Expressions & Equations

<table>
<thead>
<tr>
<th>C07</th>
<th>Solve logarithmic equations by rewriting in exponential form, including applications.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C08</td>
<td>Expand a logarithm into a linear combination of simple logarithms.</td>
</tr>
<tr>
<td>C09</td>
<td>Solve logarithmic equations by applying the One-To-One Property.</td>
</tr>
<tr>
<td>C10</td>
<td>Evaluate logarithmic expressions by applying properties of logarithms.</td>
</tr>
</tbody>
</table>

### Category D: Rational Expressions & Equations

<table>
<thead>
<tr>
<th>D01</th>
<th>Graph a rational function by finding or creating the following: asymptotes, end and local behaviors, removable discontinuity, and key points.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D02</td>
<td>Find vertical and horizontal asymptotes of rational functions.</td>
</tr>
<tr>
<td>D03</td>
<td>Determine the domain and range of rational functions.</td>
</tr>
<tr>
<td>D04</td>
<td>Solve rational equations using algebraic, graphic, and tabular methods.</td>
</tr>
</tbody>
</table>

### Category E: Radical Expressions & Equations

<table>
<thead>
<tr>
<th>E01</th>
<th>Create multiple representations (table, equation, graph, verbal description) of a radical function given the same function in a different format.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E02</td>
<td>Find the domain and range of radical functions.</td>
</tr>
<tr>
<td>E03</td>
<td>Find inverses of radical functions.</td>
</tr>
<tr>
<td>E04</td>
<td>Solve radical equations.</td>
</tr>
</tbody>
</table>

### Category F: Polynomial Expressions & Equations

<table>
<thead>
<tr>
<th>F01</th>
<th>Evaluate polynomial functions by substitution or by synthetic division and the Remainder Theorem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F02</td>
<td>Factor a polynomial of degree three or higher in using the Factor Theorem.</td>
</tr>
<tr>
<td>F03</td>
<td>Apply the Rational Root Theorem, Factor Theorem, and synthetic division to find all roots (zeros) of polynomial functions.</td>
</tr>
<tr>
<td>F04</td>
<td>Identify the x-intercepts (if applicable) and y-intercept of polynomial functions.</td>
</tr>
<tr>
<td>F05</td>
<td>Determine the equation of a polynomial function from given roots (real or complex), including in an application.</td>
</tr>
<tr>
<td>F06</td>
<td>Describe end behavior of a polynomial function.</td>
</tr>
<tr>
<td>F07</td>
<td>Solve a polynomial inequality of degree two or higher.</td>
</tr>
</tbody>
</table>
Category F: Polynomial Expressions & Equations

F08  Solve a quadratic equation by completing the square.

F09  Solve quadratic equations and applications (projectile motion, geometric area, Pythagorean Theorem, etc.) using the algebraic method of the Quadratic Formula.

F10  State the maximum number of turning points for a given polynomial function.

F11  Use the multiplicity of a zero to determine the behavior of the graph of a polynomial function at each x-intercept.

F12  Graph a polynomial function.

Category G: Conic Sections

G01  Given the equation of a circle, identify the center and radius of the circle and sketch the graph.

G02  Determine the equation of the circle given geometric information.

ENTRY-LEVEL PRECALCULUS ACES

Category A: Functions & Miscellany

A01  Perform basic operations involving complex numbers.

A02  Simplify algebraic expressions.

A03  Determine the domain and range of a relation given a table, graph, or equation.

A04  Find the composition of two functions.

A05  Identify the parent function (square, cube, square root, cube root, absolute value, reciprocal, exponential/logarithmic) and transformations (translations, reflections, dilations) for a given transformed function and use them to graph the functions.

A06  Apply polynomial, logarithmic, and exponential functions to model real life data.

A07  Algebraically determine the inverse of a function.

A08  Evaluate and graph a given piecewise function.

A09  Determine the symmetry of a relation given the graph or equation.

Category B: Systems

B01  Solve a system of linear equations.
### Category C: Exponential & Logarithmic Expressions & Equations

<table>
<thead>
<tr>
<th>Code</th>
<th>ACE Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C01</td>
<td>Solve an exponential equation (including in real world situations).</td>
</tr>
<tr>
<td>C02</td>
<td>Find the inverse of logarithmic and exponential functions.</td>
</tr>
<tr>
<td>C03</td>
<td>Expand a logarithmic expression into a linear combination of simple logarithms.</td>
</tr>
<tr>
<td>C04</td>
<td>Evaluate a logarithmic expression using properties of logarithms.</td>
</tr>
<tr>
<td>C05</td>
<td>Evaluate logarithmic equations by rewriting as exponential expressions.</td>
</tr>
<tr>
<td>C06</td>
<td>Solve logarithmic equations.</td>
</tr>
<tr>
<td>C07</td>
<td>Determine the domain and range of a logarithmic function.</td>
</tr>
</tbody>
</table>

### Category D: Rational Expressions & Equations

<table>
<thead>
<tr>
<th>Code</th>
<th>ACE Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D01</td>
<td>Solve rational inequalities.</td>
</tr>
</tbody>
</table>

### Category E: Radical Expressions & Equations

No ACEs in this category.

### Category F: Polynomial Expressions & Equations

<table>
<thead>
<tr>
<th>Code</th>
<th>ACE Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F01</td>
<td>Identify and solve equations that are quadratic in form, including application problems.</td>
</tr>
<tr>
<td>F02</td>
<td>Apply the Rational Root Theorem, the Factor Theorem, and synthetic division to find all roots (zeros) of polynomial functions.</td>
</tr>
<tr>
<td>F03</td>
<td>Factor a polynomial using the Factor Theorem.</td>
</tr>
<tr>
<td>F04</td>
<td>Evaluate a polynomial function at a specified number by using synthetic division and the Remainder Theorem.</td>
</tr>
<tr>
<td>F05</td>
<td>Graph a polynomial function.</td>
</tr>
<tr>
<td>F06</td>
<td>Solve polynomial inequalities of degree two or higher.</td>
</tr>
</tbody>
</table>

### Category G: Conic Sections

No ACEs in this category.
MATHEMATICS PROFESSIONAL ALIGNMENT COUNCIL

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Judith Bender, Goose Creek ISD
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continued on next page
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- Spring Branch Independent School District
- Spring Independent School District
- Wharton Independent School District