ACCESS Curricula Guide for

MATHEMATICS

Algebra II/Intermediate Algebra,
Precalculus, and (Non-STEM) Statistics
with Embedded Real World Applications
in Assignments and Assessments
ACCESS Curricula Guide for MATHEMATICS

Aligning Curricula and Career Education for Student Success (ACCESS)

Algebra II/Intermediate Algebra, Precalculus, and (Non-STEM) Statistics with Embedded Real World Applications in Assignments and Assessments

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INTRODUCTION

The ACCESS Curricula Guide for Mathematics is the result of many passionate discussions about Intermediate Algebra (Algebra II), Precalculus, and non-STEM Statistics among more than 100 K-12 and postsecondary math faculty and Career Technical Education (CTE) instructors throughout California. These courses were chosen for discussion because data on math course enrollment at a number of K-12s, community colleges, and universities across California indicated that large numbers of students enroll in them, often to complete prerequisites for other math courses offered at those institutions. The goal of these discussions was to define the exit and entrance competencies that students should possess in these three math courses. CTE instructors worked with math faculty to develop assessments that teachers can use to measure whether students are meeting expectations, and make these assessments apply to real world experiences.

This effort, called ACCESS (Aligning Curricula and Career Education for Student Success), launched in September 2008 with the backing of foundation leaders who believe in the type of intersegmental work that distinguishes Cal-PASS from other educational initiatives. The foundations are: The William and Flora Hewlett Foundation, The James Irvine Foundation, The Evelyn and Walter Haas Jr. Fund, and The Girard Foundation.

Thirteen regions around the state played a role in defining core competencies in Algebra II, Precalculus, and non-STEM Statistics, and developing assessments to measure achievement in each course. Based on the experience of the math faculty and their knowledge of the California State Standards and Community College Student Learning Outcomes, topics and skills were identified that the faculty felt were important for students to know upon completing the courses. Once these important topics and skills

This guide is unique in three distinct ways:

1) It takes an important step beyond listing the gaps between high school and college math skills by providing sample assessments that measure mastery of the core competencies.

2) It is the result of educational segments working in unison.

3) It answers the often-asked question from students: “Where will I ever have to use this?”
were identified and worded as measurable competencies, they were mapped relative to the California Content Standards and Common Core Standards for alignment. (See Appendix: Mapping ACCESS Competencies to California Content Standards and Common Core Standards, page 216.)

**The Problem**

California high school graduates leave high school believing they are ready for college, but the data demonstrate that even successful high school students are often ill-prepared for college. Research over the past 10 years points to specific areas that are needed — both in class and in the home — to prepare students for the rigors and habits needed to succeed in college and beyond.

The messages most California high school students receive about standards for attending a broad-access university or open-access community college are confusing. Because it is generally perceived that it is easy to enter the community college and California State University (CSU) systems, there are few intrinsic incentives to work hard in high school. Once students enroll in these colleges, however, they face challenging placement exams, faculty and university expectations, and graduation requirements of which they are likely unaware.

Remediation rates in college are staggering: 53 percent of students matriculating into the CSU system and as high as 90 percent at some community colleges.†

Unlike messages students receive from competitive four-year universities, messages received by students aiming for what are perceived as less-selective universities provide little information about the educational level at which they should achieve. This lack of information is represented by the number of college freshmen who must remediate in math and/or English and those who drop out.

Remediation rates in college are staggering: 53 percent of students matriculating into the CSU system and as high as 90 percent at some community colleges.† While a majority of high school graduates enter college, fewer than half leave with a degree. Many factors influence this attrition,

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but a report by the American Diploma Project states that the preparation students receive in high school has been found to be the greatest predictor of bachelor’s degree attainment.‡

Although remediation rates at postsecondary institutions have remained steady for 30 years, the population going to college is rising, and the number of students needing remediation is putting a strain on budgets and making the college experience longer and more arduous. Students who are unprepared are often unaware of this fact until they matriculate as freshmen. Already accepted or registered, they are given placement tests to determine if they are ready for college-level work. The CSU accepts students who, by all indicators, are ready (have taken necessary college-prep courses), but once at the university, these students must contend with their lack of preparedness for college-level work. Community colleges accept all students, regardless of courses taken in high school.

The main source of this disconnect is the lack of communication and collaboration between high school and higher education.§ High school teachers and college professors rarely talk to each other about curriculum, learning issues, and expectations. This leads to confusion by high school students and administrators regarding what it means to be prepared for college.

**Heading Toward a Solution**

With specific instructions from ACCESS project managers, 13 Cal-PASS math Professional Learning Councils (regional councils made up of teams of discipline-based faculty from elementary, middle school, high school, community college, and university segments) from around California deconstructed what they teach in the classroom. Conversation was not always smooth, as opinions surfaced about what students really need to know in Algebra II, Precalculus, and (non-STEM) Statistics to prepare them for college and careers.

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Research and documents (like those from the math deconstruction project led by Cal-PASS participants**) ignited passionate discourse among math faculty from high schools, community colleges, and universities in California. Gathering math faculty from across educational segments is something Cal-PASS does often, and is a feat unto itself. The vision of this project was already realized when 200 math and English teachers gathered for three days to debate, challenge, argue, and — finally — come to consensus. Cal-PASS groups worked tirelessly to articulate what students should know when they walk into, and then out of, math and English classes from 11th grade to transfer-level at community college.

The vision of this project was already realized when 200 English and math teachers gathered for three days to debate, challenge, argue, and — finally — come to consensus.

Under the ACCESS grant, Cal-PASS Professional Learning Councils worked throughout the second year of the grant to refine and align the competencies and begin the work of designing assessments for each competency. They designed these assessments specifically to be of high interest to students and to apply to real world work situations with the intention of linking math to careers (contextualized learning).

During the summer of 2010, a convening was held for both teachers and CTE instructors to embed CTE examples and assessments into the competencies. They used the career clusters provided by the California Community Colleges Chancellor’s Office to guide them in this CTE crossover effort.

The Final Product
This ACCESS Curricula Guide for Mathematics is the summation of math faculty ACCESS work and represents newly aligned, contextually relevant, and collaboratively developed curricula. This document is intended to supplement — not supplant — statewide or institution-based curricula and expectations for multiple audiences, including high school and postsecondary faculty, curriculum and instruction professionals, and campus-level or statewide educational leaders.

** Cal-PASS, Algebra I California Content Standards, Standards Deconstruction Project v. 2.0, 2008; Algebra II California Content Standards, Standards Deconstruction Project v. 2.0, 2008; Geometry California Content Standards, Standards Deconstruction Project v. 1.0, 2008; Pre-Calculus California Content Standards, Standards Deconstruction Project v. 1.0, 2008. www.cal-pass.org/Councils.aspx.
Specifically, this guide

1. documents the necessary competencies of three subsequent levels of math (spanning from high school to college);

2. provides assessment examples for how faculty can evaluate student competency levels;

3. embeds real world applications into assignments and assessments to contextualize materials outside of academia; and

4. maps the relationship between ACCESS curricula and California-based and nationally based standards.

It is our hope that K-12 teachers, as well as math instructors at community colleges and universities, will find the curricula and assessments in this guide both user-friendly and useful. Further, those in math teacher preparation programs could use this guide as a textbook supplement to provide concrete examples of what should be practiced regularly in the classroom to help students prepare for college and careers.
This graphic served as a primary source of discussion during the two summer ACCESS convenings of math and English faculty. It guided groups in determining how best to assess students using real world applications.
HOW TO USE THIS GUIDE

The intended use of the ACCESS Curricula Guide for Mathematics is to be a supplement to course materials and lesson plans. Math faculty from around the state identified competencies in Algebra II, Precalculus, and (non-STEM) Statistics that they believe are essential to the goals of college attainment and success. The competencies are not necessarily organized in the same way as the California State Standards or Common Core Standards for Algebra II and Precalculus, nor are they necessarily in the order in which one would teach them in the course.

Each of the competencies includes one or more model assessment items. In no way are these model assessment items exhaustive, but they are intended to indicate the level of knowledge and skills expected of the students.

Many competencies include real world example assessments. The provided real world assessments are just samples; they do not address all possibilities in terms of the CTE pathways. The purpose of including real world example assessments was to give the reader examples of how the competencies might apply to various professions. Real world assessments are not included with every competency; some of the competencies are prerequisites for the next level of skills or courses and do not lend themselves to real world examples. Real world assessments are included where appropriate.

One way to use this guide is to first identify the topic in which you are interested and locate the corresponding competency. Readers also can look at the corresponding California State or Common Core Standard to identify the relevant competency, and then use the model assessment items to gauge the level of understanding expected of students. Teachers should feel free to modify these assessment items as they see fit.
List of ACCESS Math Competencies

Algebra II/Intermediate Algebra ACCESS Competencies

1. Use properties of absolute values to solve multistep equations and inequalities.
2. Solve systems of linear equations in both two and three variables by substitution, elimination/addition, graphs, and matrices.
3. Factor polynomials by grouping.
4. Factor polynomials by using the sum/difference of cubes pattern.
5. Factor polynomials by extending the difference of squares pattern to polynomials of degree higher than 2.
6. Solve quadratic equations in the real and complex number systems by factoring.
7. Solve quadratic equations in the real and complex number systems by completing the square.
8. Solve quadratic equations in the real and complex number systems by using the quadratic formula.
9. Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in quadratic equations.
10. Add and subtract complex numbers.
11. Multiply complex numbers.
12. Perform division with complex numbers.
13. Simplify rational expressions with higher order polynomials in the denominator by canceling common factors in the numerator and denominator.
14. Add and subtract rational expressions with higher order polynomials in the denominator.
15. Multiply and divide rational expressions with higher order polynomials in the denominator.
16. Divide polynomials by binomials using long division and synthetic division.
17. Simplify a fraction where the numerator, denominator, or both contain a fraction (complex fractions).
18. Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in equations involving rational expressions.
19. Use the properties of exponents to simplify expressions with rational exponents.
20. Use the properties of exponents to solve equations with rational exponents.
21. Simplify radical expressions by removing repeated factors and performing addition and subtraction operations.
22. Simplify radical expressions by removing repeated factors and performing multiplication operations.

23. Rationalize the denominator of radical expressions.

24. Use multiple representations (tables, graphs, and equations) to solve problems that result in radical equations.

25. Apply and graph transformations of parent functions (quadratic, cubic, square root, absolute value, exponential, and logarithmic).

26. Identify the parent function (quadratic, cubic, square root, absolute value, exponential, and logarithmic) and transformations for a given transformed function in algebraic or graphical form.

27. Compose two linear, quadratic, cubic, square root, absolute value, exponential, or logarithmic functions.

28. Find the inverse of linear, quadratic, cubic, square root, absolute value, exponential, and logarithmic functions algebraically and graphically.

29. Find the equation of an exponential function given two points on the graph.

30. Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in one- and two-step exponential and logarithmic equations.

31. Graph simple conics, including:
   - a circle given its equation (centered anywhere)
   - an ellipse given its equation (centered at the origin)
   - a hyperbola given its equation (centered at the origin)

32. Solve statistical problems:
   - Compute permutations using fundamental counting principle.
   - Compute combinations.
   - Compute probabilities using combinations.
   - Compute probabilities using permutations.
   - Expand binomial expressions using the binomial theorem.

33. Compute the general term and sums of arithmetic series, finite geometric series, and infinite geometric series.
Precalculus ACCESS Competencies

1. Perform matrix operations, including addition and multiplication, and calculate the determinants of 2x2 and 3x3 matrices.
2. Describe different types of functions (polynomial, exponential, logarithmic, and trigonometric) using a table, a graph, an equation, and a verbal description.
3. Graph piecewise-defined functions.
4. Find all real zeros (roots) of polynomial functions exactly using the rational root theorem.
5. Find all the zeros (roots) of a polynomial function.
7. Solve rational inequalities.
8. Find the composition of two or more functions, each containing two or more operations (such as linear, quadratic, rational, cubic, square root, cube root, and trigonometric functions).
9. Given a function, determine if an inverse function exists. If so:
   • Find the inverse of the function algebraically and graphically.
   • Identify the domain and range of the original and inverse functions.
10. Simplify expressions involving exponents.
11. Expand logarithmic expressions.
12. Condense logarithmic expressions.
13. Solve multistep exponential equations.
15. Sketch the graph of a conic, identifying center, vertices, foci, and asymptotes (if present).
16. Graph trigonometric functions of the form \( y = A f(Bx + C) + k \).
   • Determine the amplitude of a sine or cosine function from its equation.
   • Determine the period of a sine, cosine, or tangent function from its equation.
   • Determine the vertical shift of a sine, cosine, or tangent function from its equation.
   • Determine the phase shift of a sine, cosine, or tangent function from its equation.
   • Sketch by hand the graph of a sine, cosine, or tangent function.
17. Given the graph of a trigonometric function, write the equation (sine, cosine, tangent).
18. Graph inverse trigonometric functions.
19. Prove trigonometric identities.
20. Solve trigonometry equations.
21. Solve for all unknown parts of a given triangle:
   • by using right triangle trigonometry
   • by using the Law of Sines
   • by using the Law of Cosines

22. Convert between polar and rectangular coordinates.

23. Graph equations written in polar form.

24. Graph equations written in parametric form.

25. Given the components of a two-dimensional vector, determine the magnitude and direction of the vector.

26. Perform the vector operations of addition, subtraction, and scalar multiplication.

27. Use factorial notation.

28. Expand binomial expressions.
Statistics (Non-STEM) ACCESS Competencies

1. Determine whether a variable is categorical or quantitative, given a situation.
2. Identify the sampling method used (random, systematic, convenience, stratified, cluster, or voluntary response), given a scenario in which data were collected from a population.
3. Identify the basic terminology of experimental design used in a given scenario.
4. Identify biases in experimental designs or in the creation of a sample.
5. Determine an appropriate experimental design for a given scenario.
6. Construct and describe an appropriate visual display given a data set (histogram, bar graph, stem plot, scatter plot, box and whisker plot, and pie chart).
7. Identify which measure of the center (mean, median, or mode) is most appropriate for a given situation and what the relationship between the mean, median, and mode indicates about the distribution of the data.
8. Determine standard deviation from multiple representations of data (table, graph, and word problems) and explain standard deviation in the context of a given situation.
9. Use the normal distribution to solve for the probability of an event or solve for the boundaries for a particular event.
10. Determine the size of a sample space using counting principles, permutations, and combinations.
11. Compute basic probability of independent events and solve problems involving the binomial distribution:
   - Determine situations that are Bernoulli trials.
   - Calculate the probability of an event (success) and its complementary probability (failure).
   - Discern the difference between geometric probability and binomial probability of Bernoulli trials.
   - Apply the formula for binomial probability for \( x \) successes in \( n \) trials
     \[
P(x) = \binom{n}{x} p^x q^{n-x}.
\]
   - Use the normal model \( N(\mu, \sigma) \), where \( \mu = np \) and \( \sigma = \sqrt{npq} \) to approximate a binomial probability when the number of trials and desired successes is inordinately large.
12. Apply the Central Limit Theorem to sampling distributions.
13. Estimate population parameters using confidence intervals for means and proportions.
14. Perform appropriate hypothesis test (state the hypotheses, determine the significance level, check conditions/criteria, calculate the test statistic, determine the $p$-value, make a decision and interpret it in the context of the problem) for a given situation.

15. Explain the relationship between parameters and statistics.

16. Given a $p$-value, interpret its meaning in the context of the variables of a problem:
   - Define the $p$-value.
   - Identify the appropriate null and alternative hypotheses, and use the proper notation of $H_0$ and $H_a$.
   - Interpret a given $p$-value in the context of a hypothesis testing situation as it relates to the rejection of or failure to reject $H_0$.

17. Find and apply the equation of the regression line when linear regression is appropriate:
   - Determine when linear regression is appropriate for a set of bivariate data.
   - Find the linear correlation coefficient $r$ and the coefficient of determination $r^2$.
   - Use the regression line to make appropriate predictions.
   - Interpret $r$, $r^2$, and slope in the context of a given situation.
Algebra II/Intermediate Algebra Competencies
COMPETENCY 1

Use properties of absolute values to solve multistep equations and inequalities.

Prior Knowledge:

- Solve algebraic equations.
- Solve algebraic inequalities.
- Evaluate absolute value expressions for a given value of the variable.
- Graph solutions of an inequality on the real number line.
- Interpret the absolute value of a number as a distance from zero on a number line.
- Represent solution sets using inequalities.
- Convert solution sets written in interval notation to inequality notation and vice versa.
- Solve compound inequalities.
- Translate absolute value inequalities into compound inequalities.
- Identify absolute value equations that have no solution (e.g., \(|x - 3| = -9\)).
- Identify absolute value inequalities that have no solution (e.g., \(|x + 2| < 0\)).

Model Assessments

Assessment Example 1:

A person’s body temperature, \(t\), is considered normal if it falls within 1.5°F of 98.6°F. Write an absolute value inequality to express this fact.

*Model Assessment Answer(s)*:

\(|t - 98.6| \leq 1.5.\)

Assessment Example 2:

Express the fact that \(x\) differs from -3 by more than 5 as an inequality involving an absolute value. Solve the inequality for \(x\).

*Model Assessment Answer(s)*:

\(|x + 3| > 5.\)

Solution: \(x < -8\) or \(x > 2.\)
Assessment Example 3:
Solve for \( x \): \(-2|3x - 5| + 1 < -7\).

*Model Assessment Answer(s):*
\[ x > 3 \text{or} x < \frac{1}{3}. \]

Assessment Example 4:
a) A construction company is building houses. The framing crew needs to place studs 3 feet apart with a tolerance of a quarter inch to pass inspection. Write an absolute value inequality for this solution set and solve. Explain your reasoning to the foreman of the project.

b) ABC Manufacturing makes doors and windows. Each door must be 42 inches in width with a tolerance of half an inch either way in order to fit in the frame. Write an inequality to solve this problem. Explain your reasoning. If doors are 84 inches tall, what is the difference in materials that would be necessary to construct 1,000 doors at the two extremes?

*Model Assessment Answer(s):*
a) Let \( x \) be the distance between the studs. Then the inequality that must be solved is
\[ |x - 36| \leq \frac{1}{4}, \]
and the solution set is
\[ 35 \frac{3}{4} \leq x \leq 36 \frac{1}{4}, \]
so the distance between the studs must be at least 35 inches but no more than 36 inches.
b) Let $x$ be the actual width of the door. Then the inequality that must be solved is

$$|x - 42| \leq \frac{1}{2},$$

and the solution set is

$$\frac{83}{2} \leq x \leq \frac{85}{2},$$

so the door's width must measure at least 41½ inches but no more than 42½ inches. If we let $y$ be the height of the doors, then the inequality is

$$|y - 84| \leq \frac{1}{2},$$

and the solution set is

$$\frac{167}{2} \leq y \leq \frac{169}{2},$$

so the door's height must measure at least 83½ inches but no more than 84½ inches. The amount of material in the smallest allowable door would be

$$41\frac{1}{2} \text{ in.} \times 83\frac{1}{2} \text{ in.} = 3,465.25 \text{ sq. in. (about 24.064 sq. ft.).}$$

The material in the largest allowable door would be

$$42\frac{1}{2} \text{ in.} \times 84\frac{1}{2} \text{ in.} = 3,591.25 \text{ sq. in. (about 24.939 sq. ft.).}$$

The difference is 0.875 square feet, so multiplying that by 1,000 doors would be 875 square feet of additional material if all of the doors are as large as possible rather than as small as possible.
COMPETENCY 2

Solve systems of linear equations in both two and three variables by substitution, elimination/addition, graphs, and matrices.

Prior Knowledge:

• Solve and graph a linear equation in two variables.
• Solve and graph a linear inequality in two variables.
• Substitute a rational number or expression for a variable.
• Identify the coefficients from an equation in standard form.

Model Assessments

Assessment Example 1:

A popular new band will be performing at a local concert venue. There are 800 tickets available. Tickets for reserved seats cost $100, general admission tickets cost $70, and tickets for lawn seating cost $50. If all the lawn and reserved seats and half the general admission seats are sold, the total collected would be $49,000. If all the tickets are sold, the total would be $63,000. How many seats of each type are available at the Pavilion?

Model Assessment Answer(s):

Let $x$ = number of reserved seat tickets, $y$ = number of general admission tickets, and $z$ = number of lawn seating tickets. The system of equations that must be solved is:

\[
\begin{align*}
800 &= x + y + z \\
63000 &= 100x + 70y + 50z \\
49000 &= 100x + 70(0.5y) + 50z
\end{align*}
\]

Solution: 300 reserved, 400 general, and 100 lawn seats.

Assessment Example 2:

Solve using substitution:

\[
\begin{align*}
-x + 2y &= 11 \\
3x - 2y &= -13
\end{align*}
\]

Model Assessment Answer(s):

\((-1, 5)\).
Assessment Example 3:

Solve using matrices:

\[\begin{align*}
\begin{bmatrix}
1 & -2 & 3 & 2 \\
-1 & -5 & 2 & -11 \\
2 & -1 & -4 & 0
\end{bmatrix}
\end{align*}\]

Write the augmented matrix for the above system of equations. Use Gaussian elimination to transform the augmented matrix into row-echelon form (by hand) or use the Gauss-Jordan method to transform the augmented matrix into reduced row-echelon form (by hand or using a calculator). The augmented matrix and reduced row-echelon form (RREF) of the system of equations is:

\[\begin{align*}
A &= \begin{bmatrix}
1 & -2 & 3 & 2 \\
-1 & -5 & 2 & -11 \\
2 & -1 & -4 & 0
\end{bmatrix},
\text{and } \text{rref}(A) &= \begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 1
\end{bmatrix}
\end{align*}\]

Answer: \(x = 3, y = 2, z = 1\). Solution should be written as an ordered triple: \((3, 2, 1)\).

Assessment Example 4:

Solve the system using the graphing method:

\[\begin{align*}
\begin{cases}
x + y &= 3, \\
x - y &= -5.
\end{cases}
\end{align*}\]

Model Assessment Answer(s):

\((4, -1)\).
Assessment Example 5:

a) Forever Bloom Florist is placing an order for roses, lilies, and tulips. At the nearby wholesale flower market, 4 dozen lilies, 3 dozen roses, and 2 dozen tulips cost $124; 1 dozen lilies, 2 dozen roses, and 3 dozen tulips cost $76; 2 dozen lilies, 4 dozen roses, and 3 dozen tulips cost $128. How much does each individual type of flower cost per dozen?

b) You have $12,800 to invest in a 401K. You plan to diversify your money between a money market account, an income fund, and a growth fund. You have decided to put twice as much money in the growth fund as the money market account in order to maximize your potential earnings. The growth fund earns 10% interest, the income fund earns 7%, and the money market account earns 5%. How much should you invest in each account to earn $1,000 annually in simple interest?

Model Assessment Answer(s):

a) Let $x$ = cost per dozen roses, $y$ = cost per dozen lilies, and $z$ = cost per dozen tulips. The system of equations that must be solved is:

$$3x + 4y + 2z = 124,$$
$$2x + y + 3z = 76,$$
$$4x + 2y + 3z = 128.$$

The cost per dozen roses is $20; the cost per dozen lilies is $12, and the cost per dozen tulips is $8.

b) Let $x$ = the amount of money in the money market account, $y$ = the amount of money in the income fund, and $z$ = the amount of money in the growth fund. The system of equations that must be solved is:

$$x + y + z = 12800,$$
$$2x - z = 0,$$
$$0.05x + 0.07y + 0.10x = 1000.$$

Put $2,600 in the money market account, $5,000 in the income fund, and $5,200 in the growth fund.
COMPETENCY 3

Factor polynomials by grouping. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:
- Apply the Laws of Exponents to algebraic expressions.
- Recognize numbers as being perfect squares or perfect cubes.
- Recognize binomials as the difference of two square terms.
- Factor out a common monomial from a polynomial.

Model Assessments

Assessment Example 1:
Factor by grouping: $2x^2 + 4x + 3x + 6$.

Model Assessment Answer(s):
$(2x + 3)(x + 2)$.

Assessment Example 2:
Factor by grouping: $x^4 - x^3 + x - 1$.

Model Assessment Answer(s):
$(x + 1)(x - 1)(x^2 - x + 1)$. 
COMPETENCY 4

Factor polynomials by using the sum/difference of cubes pattern.

Prior Knowledge:

• Apply the Laws of Exponents to algebraic expressions.
• Recognize numbers as being perfect squares or perfect cubes.
• Recognize binomials as the difference of two square terms.
• Factor a second-degree polynomial as the product of two binomials using standard factoring techniques.
• Factor out a common monomial from a polynomial.
• Recognize that a polynomial is a perfect square trinomial.

Model Assessments

Assessment Example 1:

A manufacturing company orders 5 ft. x 5 ft. x 5 ft. foam cubes. The company customizes smaller cubes based on their customer’s order. However, to save on waste, they want to sell the remaining three foam blocks and need to have the dimensions listed for their Web site. What are the dimensions of the three foam blocks?

Model Assessment Answer(s):

Amount remaining is \(5^3 - a^3\), where \(a\) is the side length of the ordered cube.

When factored using the difference of squares pattern, the result is \((5 - a)(25 + 5a + a^2)\).

By distributing, the size of each rectangular prism is as follows:

The first piece is \((5 - a)(25)\) or \((5 - a)\) ft. \(\times\) 5 ft. \(\times\) 5 ft.

The second piece is \((5 - a)5a\) or \((5 - a)\) ft. \(\times\) 5 ft. \(\times\) a ft.

The third piece is \((5 - a)a^2\) or \((5 - a)\) ft. \(\times\) a ft. \(\times\) a ft.

Real World Application Reference:

• Engineering and Design
• Manufacturing and Product Development
Assessment Example 2:
Factor: $125x^3 + 64$.

*Model Assessment Answer(s):*

$(5x + 4)(25x^2 - 20x + 16)$.

Assessment Example 3:
Factor: $27x^3 - 8$.

*Model Assessment Answer(s):*

$(3x - 2)(9x^2 + 6x + 4)$.

Assessment Example 4:
Factor: $(3x - 2)^3 - 27$.

*Model Assessment Answer(s):*

$(3x - 5)(9x^2 - 3x + 7)$. 
**COMPETENCY 5**

Factor polynomials by extending the difference of squares pattern to polynomials of degree higher than 2. (This a prerequisite skill that is used in Precalculus and Calculus.)

**Prior Knowledge:**
- Apply standard factoring techniques to factoring second degree polynomials.

**Model Assessments**

**Assessment Example 1:**
Factor: \(x^4 - 81\).

*Model Assessment Answer(s):*
\[(x + 3)(x - 3)(x^2 + 9).\]

**Assessment Example 2:**
Factor: \((x - 5)^2 - 16y^4\).

*Model Assessment Answer(s):*
\[(x - 5 - 4y^2)(x - 5 + 4y^2).\]
COMPETENCY 6

Solve quadratic equations in the real and complex number systems by factoring.

Prior Knowledge:

- Factor quadratic expressions.
- Simplify radicals.
- Solve a quadratic equation using factoring, completing the square, or the quadratic formula.
- Simplify a radical when the radicand is negative.
- Find the roots of an equation.
- Find the x- and y-intercepts of an equation.
- Find the vertex of a parabola.
- Determine if a parabola will open upward or downward.

Model Assessments

Assessment Example 1:

Your construction company has just contracted with the owners of a condemned building downtown to demolish the present building and erect a new one in its place. You have 300 meters of fencing available to enclose the building and surrounding area, which is 5,000 square meters. Find the dimensions of the rectangular area that can be encompassed by the available fencing.

Model Assessment Answer(s):

Write the perimeter equation $2L + 2W = 300$ and solve for $L$. $L = (300 - 2W) / 2$. Substitute this into the area equation $A = W \left( \frac{300 - 2W}{2} \right) = 5000$.

Solve the polynomial equation $W^2 - 150W + 5000 = 0$. Solution: 50 m. x 100 m.

Assessment Example 2:

Solve: $x^3 - x^2 = 1 - x$.

Model Assessment Answer(s):

$x = \pm i$ or $x = 1$. 

Real World Application Reference:

- Building Trades and Construction
COMPETENCY 7

Solve quadratic equations in the real and complex number systems by completing the square.

Prior Knowledge:
- Factor quadratic expressions.
- Simplify radicals.
- Solve a quadratic equation using factoring, completing the square, or the quadratic formula.
- Simplify a radical when the radicand is negative.
- Find the roots of an equation.
- Find the x- and y-intercepts of an equation.
- Find the vertex of a parabola.
- Determine if a parabola will open up or down.

Model Assessments

Assessment Example 1:
A design company has decided it is aesthetically pleasing from an aerial view if the area of a pool is equal to the area of the border around the pool. Given a pool that is 6 meters wide and 10 meters long that is surrounded by a border that is of uniform width, determine the width of the border.

Model Assessment Answer(s):

\[-4 + \sqrt{31} \approx 1.57 \text{ m.}\]

Assessment Example 2:
Solve the equation by completing the square: \(x^2 - 6x = 13\).

Model Assessment Answer(s):

\[x = 3 \pm \sqrt{22}.\]

Assessment Example 3:
Solve: \(x^2 + 4x = -20\).

Model Assessment Answer(s):

\[x = -2 \pm 4i.\]
COMPETENCY 8

Solve quadratic equations in the real and complex number systems by using the quadratic formula.

Prior Knowledge:
- Factor quadratic expressions.
- Simplify radicals.
- Solve a quadratic equation using factoring, completing the square, or the quadratic formula.
- Simplify a radical when the radicand is negative.
- Find the roots of an equation.
- Find the x- and y-intercepts of an equation.
- Find the vertex of a parabola.
- Determine if a parabola will open up or down.

Model Assessments

Assessment Example 1:
Robin, a Calculus student at the local university, throws her Calculus book out of her dormitory window straight up into the air with a velocity of 48 feet per second. Assuming that her dormitory room is 40 feet above the ground, and ignoring air resistance, the height of her book after t seconds will be: \( s(t) = -16t^2 + 48t + 40 \). How long does it take before the book hits the foot of the person standing directly below her window?

Model Assessment Answer(s):
\[ t = 3.7 \text{ seconds}. \]

Assessment Example 2:
Solve: \( x^2 - 4x - 1 = 0 \).

Model Assessment Answer(s):
\[ 2 \pm \sqrt{5}. \]
**Assessment Example 3:**

Solve: \( 4 - \frac{1}{x} - \frac{2}{x^2} = 0. \)

*Model Assessment Answer(s):*

Even though the equation is a rational equation and not technically a quadratic equation, once both sides of the equation are multiplied by the least common denominator, the resulting equation is quadratic and can be solved.

\[
\frac{1 \pm \sqrt{33}}{8}.
\]

**Assessment Example 4:**

Solve: \( 2x^2 + 5x = -11. \)

*Model Assessment Answer(s):*

\[
x = \frac{-5 \pm 3\sqrt{7}i}{4} \quad \text{or} \quad x = \frac{-5 \pm 3\sqrt{7}}{4}.
\]
COMPETENCY 9

Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in quadratic equations.

Prior Knowledge:

- Factor quadratic expressions.
- Simplify radicals.
- Solve a quadratic equation using factoring, completing the square, or the quadratic formula.
- Simplify a radical when the radicand is negative.
- Find the roots of an equation.
- Find the x- and y-intercepts of an equation.
- Find the vertex of a parabola.
- Determine if a parabola will open up or down.
- Recognize that the graph of any quadratic equation is a parabola.
- Recognize what the graph of $y = x^2$ looks like.

Model Assessments

Assessment Example 1:

A toy company is developing a water balloon launcher. When the balloon is shot straight up, the height of the balloon at any time $t$ (measured in seconds) can be described by the equation

$$h(t) = -16t^2 + v_0t + h_0,$$

where $v_0$ represents the initial velocity and $h_0$ represents the initial height of the water balloon. If you know that the initial velocity is 48 feet per second and the balloon is launched from a point 64 feet off the ground:

a) Write the equation that describes the height at any time.

b) Determine the height of the balloon from the ground at $t = 1$ second, then 2 seconds, etc.; create a table showing the corresponding heights.

c) Determine the time it takes for the balloon to reach maximum height.

d) Determine the time at which the balloon hits the ground. If your friend is talking on his phone directly below the launch area and it takes him 5 seconds to realize you have launched a water balloon, will it hit him before he has time to move?

e) Graph the path of the water balloon’s height with respect to time.

Real World Application Reference:

- Manufacturing and Product Development
Model Assessment Answer(s):

a) $h(t) = -16t^2 + 48t + 64$.

b) See the table below.

<table>
<thead>
<tr>
<th>Time (secs)</th>
<th>Height $h(t)$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>2</td>
<td>84</td>
</tr>
<tr>
<td>3</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>96</td>
</tr>
<tr>
<td>6</td>
<td>84</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
</tr>
<tr>
<td>8</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>-44</td>
</tr>
<tr>
<td>11</td>
<td>-96</td>
</tr>
</tbody>
</table>

c) The balloon reached a maximum height of 100 feet at 1.5 seconds after it was launched.

d) It will take 4 seconds for the balloon to hit the ground. The friend will get wet.

e) Graph:
COMPETENCY 10

Add and subtract complex numbers.

Prior Knowledge:
- Add and subtract algebraic expressions involving variables.
- Use the distributive property in algebraic expressions involving variables.
- Simplify algebraic expressions involving positive integer exponents.
- Simplify square roots of integers.
- Multiply binomials.
- Rationalize a monomial or binomial denominator.
- Identify conjugates and calculate their products.
- Multiply two numbers with exponents having the same base.

Model Assessments

Assessment Example 1:
In an AC circuit with two parallel pathways, the total impedance $Z$, in ohms, satisfies the formula

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2},$$

where $Z_1$ is the impedance of the first pathway and $Z_2$ is the impedance of the second pathway. Determine the total impedance if the impedances of the two pathways are $Z_1 = 1 + i$ and $Z_2 = 2 - 3i$.

Model Assessment Answer(s):

$$\frac{17}{13} + \frac{7}{13}i$$ ohms.

Assessment Example 2:
Simplify: $(8 + 2i) + (4 - i)$.

Model Assessment Answer(s):

$12 + i$.

Assessment Example 3:
Simplify: $(2 - 5i) - (8 + 6i)$.

Model Assessment Answer(s):

$-6 - 11i$.

Real World Application Reference:
- Energy and Utilities
- Engineering and Design
COMPETENCY 11

Multiply complex numbers.

Prior Knowledge:

- Add and subtract algebraic expressions involving variables.
- Use the distributive property in algebraic expressions involving variables.
- Simplify algebraic expressions involving positive integer exponents.
- Simplify square roots of integers.
- Multiply binomials.
- Rationalize a monomial or binomial denominator.
- Identify conjugates and calculate their products.
- Multiply two numbers with exponents having the same base.

Model Assessments

Assessment Example 1:

In an electrical circuit, the voltage ($E$) in volts, the current ($I$) in amps, and the opposition to the flow of current — called impedance — ($Z$) in ohms are related by the equation $E = IZ$. A circuit has a current of $(3 + 2i)$ amps and an impedance of $(6 - 4i)$ ohms. Determine the voltage.

Model Assessment Answer(s):

\[ E = (3 + 2i)(6 - 4i) = 26 \text{ volts.} \]

Assessment Example 2:

Multiply and combine like terms: $(2i + 3)(4 - 5i) + (-2i)^5$.

Model Assessment Answer(s):

\[ 22 - 39i. \]
Assessment Example 3:

Simplify: \( i(7 - 3i)(2 + 10i) \).

*Model Assessment Answer(s):*

\(-64 + 44i\).

Assessment Example 4:

Simplify: \( \left( \frac{\sqrt{2}}{2} + \frac{3}{4}i \right)^2 \).

*Model Assessment Answer(s):*

\(-\frac{1}{16} + \frac{3\sqrt{2}}{4}i\).
COMPETENCY 12

Perform division with complex numbers.

Prior Knowledge:
- Add and subtract algebraic expressions involving variables.
- Use the distributive property in algebraic expressions involving variables.
- Simplify algebraic expressions involving positive integer exponents.
- Simplify square roots of integers.
- Multiply binomials.
- Rationalize a monomial or binomial denominator.
- Identify conjugates and calculate their products.
- Multiply two numbers with exponents having the same base.

Model Assessments

Assessment Example 1:
Electrical circuits can be connected in series, one after another, or in parallel circuits that branch off a main line. If the circuits are hooked up in parallel, the total impedance is given by:

\[ R = \frac{R_1 R_2}{R_1 + R_2}. \]

Find the total impedance connected in parallel if the impedance of the first resistor is \( R_1 = 5 + 2i \) ohms and the impedance of the second resistor is \( R_2 = 3 - i \) ohms.

Model Assessment Answer(s):

\[
\frac{137 - 9i}{65} = \frac{9i}{65}.
\]
Assessment Example 2:

Divide to identify real and imaginary parts: \( \frac{3}{2 - i} \).

Model Assessment Answer(s):

\[
\frac{6}{5} + \frac{3}{5}i.
\]

Assessment Example 3:

Simplify: \( \frac{4 + 2i}{7 - 3i} \).

Model Assessment Answer(s):

\[
\frac{11}{29} + \frac{13}{29}i.
\]
COMPETENCY 13

Simplify rational expressions with higher order polynomials in the denominator by canceling common factors in the numerator and denominator. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:

- Use the properties of exponents.
- Add, subtract, multiply, divide, and simplify rational expressions.
- Add, subtract, multiply, and divide polynomials.
- Factor polynomials.
- Identify when a rational expression is undefined.
- Simplify a rational expression by canceling common factors in the numerator and the denominator.
- Multiply and divide rational expressions with monomial and polynomial denominators.
- Find the lowest common denominator between two or more monomial or polynomial denominators.
- Add and subtract rational expressions.
- Identify complex rational expressions.

Model Assessments

Assessment Example 1:
Simplify:

a) \( \frac{(2a^{-1}b^3)^4}{(4a^2b^{-2})^{-1}} \).

b) \( \frac{x^3 + 8}{x^3 - 3x^2 - 10x} \).

c) \( \frac{x^2 - x - 6}{x^2 - 7x + 12} \).

Model Assessment Answer(s):

a) \( \frac{64b^{10}}{a^2} \).

b) \( \frac{x^2 - 2x + 4}{x(x - 5)} \).

c) \( \frac{x + 2}{x - 4} \).
COMPETENCY 14

Add and subtract rational expressions with higher order polynomials in the denominator. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:

- Use the properties of exponents.
- Add, subtract, multiply, divide, and simplify rational expressions.
- Add, subtract, multiply, and divide polynomials.
- Factor polynomials.
- Identify when a rational expression is undefined.
- Simplify a rational expression by canceling common factors in the numerator and the denominator.
- Multiply and divide rational expressions with monomial and polynomial denominators.
- Find the lowest common denominator between two or more monomial or polynomial denominators.
- Add and subtract rational expressions.
- Identify complex rational expressions.

Model Assessments

Assessment Example 1:

Simplify:

a) \( \frac{x - 1}{x + 2} + \frac{4}{2 - x} + \frac{6x}{x^2 - 4} \)

b) \( \frac{x - 1}{x^2 + x - 6} - \frac{x - 2}{x^2 + 4x + 3} \)

c) \( \frac{3y + 2}{y - 5} + \frac{4}{3y + 4} - \frac{7y^2 + 24y + 28}{3y^2 - 11y - 20} \)

Model Assessment Answer(s):

a) \( \frac{x - 3}{x - 2} \)

b) \( \frac{4x - 5}{(x + 3)(x + 1)(x - 2)} \)

c) \( \frac{2(y + 4)}{3y + 4} \)
COMPETENCY 15

Multiply and divide rational expressions with higher order polynomials in the denominator. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:

- Use the properties of exponents.
- Add, subtract, multiply, divide, and simplify rational expressions.
- Add, subtract, multiply, and divide polynomials.
- Factor polynomials.
- Identify when a rational expression is undefined.
- Simplify a rational expression by canceling common factors in the numerator and the denominator.
- Multiply and divide rational expressions with monomial and polynomial denominators.
- Find the lowest common denominator between two or more monomial or polynomial denominators.
- Add and subtract rational expressions.
- Identify complex rational expressions.

Model Assessments

Assessment Example 1:

Simplify:

a) \[ \frac{x^2 - x - 2}{x^2 - 1} \cdot \frac{2x^2 - 5x - 3}{2x - 4} \div \frac{x^2 - 5x + 6}{2x^2 - 4x} \]

b) \[ \frac{a^2 + b}{3a^2 - 4a - 20} \cdot \frac{a^2 + 5a}{2a^2 + 11a + 5} \div \frac{ab^2}{6a^2 - 17a - 10} \]

c) \[ \frac{2x^2 - 3xy - 2y^2}{3x^2 - 4xy + y^2} \cdot \frac{3x^2 - 2xy - y^2}{x^2 + xy - 6y^2} \]

Model Assessment Answer(s):

a) \[ \frac{x(2x + 1)}{x - 1} \]

b) \[ \frac{a^2 + 1}{b(a + 2)} \]

c) \[ \frac{(2x + y)(3x + y)}{(3x - y)(x + 3y)} \]
**COMPETENCY 16**

Divide polynomials by binomials using long division and synthetic division. (This is a prerequisite skill that is used in Precalculus and Calculus.)

**Prior Knowledge:**
- Add and subtract polynomials.
- Use the distributive property to multiply polynomials.
- Apply the appropriate exponential rules to simplify algebraic expressions.
- Divide polynomials by a monomial.
- Divide polynomials by binomials using long division.
- Translate and solve multistep word problems involving polynomials.

**Model Assessments**

**Assessment Example 1:**
Use synthetic division to find the quotient of

\[
(2y^5 - 10y^3 + y - 2) ÷ (y + 1).
\]

*Model Assessment Answer(s):*

\[
2y^4 - 2y^3 - 8y^2 + 8y - 7 + \frac{5}{y + 1}.
\]

**Assessment Example 2:**
Simplify: \((12x^3 - x^2 + 4) ÷ (3x + 2)\).

*Model Assessment Answer(s):*

\[
4x^2 - 3x + 2.
\]

**Assessment Example 3:**
Simplify: \(\frac{12x^3 - 26x^2y + 19xy^2 - 15y^3}{3x - 5y}\).

*Model Assessment Answer(s):*

\[
4x^2 - 2xy + 3y^2.
\]
COMPETENCY 17

Simplify a fraction where the numerator, denominator, or both contain a fraction (complex fractions).

Prior Knowledge:

• Use the properties of exponents.
• Add, subtract, multiply, divide, and simplify rational expressions.
• Add, subtract, multiply, and divide polynomials.
• Factor polynomials.
• Identify when a rational expression is undefined.
• Simplify a rational expression by canceling common factors in the numerator and the denominator.
• Multiply and divide rational expressions with monomial and polynomial denominators.
• Find the lowest common denominator between two or more monomial or polynomial denominators.
• Add and subtract rational expressions.
• Identify complex rational expressions.

Model Assessments

Assessment Example 1:

Simplify: \( \frac{1}{x+1} + \frac{1}{x-1} \cdot \frac{x}{x} \cdot \frac{1}{x+1} \).

Model Assessment Answer(s):

\( \frac{2}{x-1} \).
Assessment Example 2:

Simplify: \[ \frac{x^{-1} + y^{-1}}{x^2 - y^2} \cdot \frac{xy}{xy} \]

*Model Assessment Answer(s):*

\[ \frac{1}{x - y} \]

Assessment Example 3:

The average rate on a round-trip delivery route that has a one-way distance of \( d \) is given by the complex rational expression

\[ \frac{2d}{\frac{d}{r_1} + \frac{d}{r_2}} \]

where \( r_1 \) and \( r_2 \) are the rates on the outgoing and return trips, respectively.

a) Simplify the complex rational expression.

b) Find the average rate if the outbound trip has an average rate of 30 mph. and the return trip has an average rate of 40 mph.

*Model Assessment Answer(s):*

a) \[ \frac{2r_1 r_2}{r_2 + r_1} \]

b) \[ \frac{2400}{70} \approx 34.3 \text{ mph.} \]
COMPETENCY 18

Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in equations involving rational expressions.

Prior Knowledge:

- Add and subtract algebraic expressions involving variables.
- Use the distributive property in algebraic expressions involving variables.
- Simplify algebraic expressions involving positive integer exponents.
- Simplify square roots of integers.
- Multiply binomials.
- Rationalize a monomial or binomial denominator.
- Identify conjugates and calculate their products.
- Multiply two numbers with exponents having the same base.

Model Assessments

Assessment Example 1:
Electrical circuits can be connected in series, one after another, or in parallel circuits that branch off a main line. If the circuits are hooked up in parallel, the reciprocal of the total resistance in the circuit is found by adding the reciprocal of each resistance, as shown below.

\[ \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}. \]

If \(R_1 = x\) and \(R_2 = x + 4\), and the total resistance \(R\) is 1.5 \(\Omega\) (ohms), find the positive value of \(R_1\).

Model Assessment Answer(s):
Solve: \(\frac{1}{x} + \frac{1}{x + 4} = \frac{1}{1.5}\); solution is \(x = 2 \ \Omega\).
Assessment Example 2:

Simplify: \( \frac{1}{4} + \frac{1}{x} = \frac{1}{5x} \).

Model Assessment Answer(s):

\[ x = -\frac{16}{5}. \]

Assessment Example 3:

Simplify: \( \frac{1}{x+4} - \frac{2x}{x+5} = \frac{-8x-10}{x^2+9x+20} \).

Model Assessment Answer(s):

\[ x = -\frac{5}{2}, 3. \]
COMPETENCY 19

Use the properties of exponents to simplify expressions with rational exponents.

Prior Knowledge:
- Apply the properties of exponents.
- Use the basic notation of fractional exponents.
- Simplify radicals.

Model Assessments

Assessment Example 1:
The half-life of aspirin in a person’s bloodstream is about 15 minutes: \( A(t) = A_0e^{\frac{t}{\tau}} \) where \( A(t) \) is the amount remaining after time \( t \), \( A_0 \) is the initial amount, and \( \tau \) is the mean lifetime. Determine the mean lifetime of aspirin in the bloodstream. (HINT: the mean lifetime, \( \tau \), is defined as half-life divided by \( \ln 2 \).) Simplify the rational exponent in the exponential equation and rewrite the equation.

Model Assessment Answer(s):
The mean lifetime, \( \tau \), is 21.64 seconds. The resulting simplified exponential equation is

\[ A(t) = A_0e^{-0.04621t}. \]

Assessment Example 2:
Neglecting overlapping regions, how much cardboard would be needed to construct an open cube-shaped box with volume \( V \)? Write an expression for the amount of material in terms of \( V \) using rational exponents.

Model Assessment Answer(s):
Let \( x \) be the length of the side of the cube. If the volume is \( V \) cubic units, then \( V = x^3 \). So \( x = V^{\frac{1}{3}} \). This means that each face of the cube has an area of \( x^2 = V^{\frac{2}{3}} \), so the open box requires \( 5V^{\frac{2}{3}} \) square units of cardboard.
Assessment Example 3:
Evaluate the following expressions:

a) \((-27)^{\frac{2}{3}}\).

b) \(16^{-\frac{3}{4}}\).

*Model Assessment Answer(s):*

a) 9.

b) \(\frac{1}{8}\).

Assessment Example 4:
Use properties of exponents to simplify \(\left(\frac{2x^{\frac{2}{3}}y^{-\frac{1}{3}}}{x^2y}\right)^5\).

*Model Assessment Answer(s):*

\[
\frac{2^5}{y^{\frac{13}{3}}} = \frac{32}{y^{\frac{13}{3}}}. 
\]
COMPETENCY 20

Use the properties of exponents to solve equations with rational exponents.

Prior Knowledge:

- Apply the properties of exponents.
- Use the basic notation of fractional exponents.
- Simplify radicals.

Model Assessments

Assessment Example 1:
Solve: $2x^3 + 3 = 19$.

Model Assessment Answer(s):

$x = 2, -1 \pm i\sqrt{3}$.

Assessment Example 2:
Solve: $5x^4 - 4 = 401$.

Model Assessment Answer(s):

$x = \pm 3, \pm 3i$.

Assessment Example 3:
The amount of material needed for an open cube-shaped box with volume $V$ is given by the expression $5\sqrt[3]{V}$. Find the cost of producing 144 boxes, each of which has a volume of 3,000 cubic centimeters, given that the cardboard costs $1.60 per square meter.

Model Assessment Answer(s):

Each box requires $5\sqrt[3]{3000} \approx 1040$ cm$^2$ of cardboard, so 144 boxes require about 150,000 square centimeters, or 15 square meters, of cardboard. The total cost is $15(1.60) = $24.00.
COMPETENCY 21

Simplify radical expressions by removing repeated factors and performing addition and subtraction operations. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:
• Use the properties of exponents.
• Simplify square roots of integers.
• Simplify algebraic expressions.

Model Assessments

Assessment Example 1:
Simplify:

a) $\sqrt{135} - \sqrt{60} - \sqrt{15}$.

b) $\sqrt{72x^3 y^2} + y\sqrt{50x^3}$.

c) $x\sqrt{54x^7} - \frac{3}{2}250x^{10}$.

d) $\frac{5}{2}32x^6 - \frac{3}{2}243x^5$.

Model Assessment Answer(s):

a) 0.

b) $11xy\sqrt{2x}$.

c) $-2x^3\sqrt{2x}$.

d) $2x^5\sqrt{x} - 3x$. 

COMPETENCY 22

Simplify radical expressions by removing repeated factors and performing multiplication operations. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:
• Use the properties of exponents.
• Simplify the square roots of integers.
• Add, subtract, and multiply polynomials.

Model Assessments

Assessment Example 1:
Find the product:

a) \((4\sqrt{2} - 3\sqrt{5})(\sqrt{5} + \sqrt{2})\).
b) \(3\sqrt{7}(2 - 6\sqrt{5})\).

Model Assessment Answer(s):

a) \(-7 + \sqrt{10}\).
b) \(6\sqrt{7} - 18\sqrt{35}\).
COMPETENCY 23

Rationalize the denominator of radical expressions. (This is a prerequisite skill that is used in Precalculus and Calculus.)

Prior Knowledge:
- Use the properties of exponents.
- Simplify square roots of integers.
- Add, subtract, and multiply polynomials.
- Add, subtract, and multiply radical expressions.

Model Assessments

Assessment Example 1:
Simplify:

a) \( \frac{12}{\sqrt{24}} \)

b) \( \frac{3}{\sqrt[3]{3x^2}} \)

c) \( \frac{2\sqrt{3} + 5}{7 - \sqrt{2}} \)

Model Assessment Answer(s):

a) \( 2\sqrt{54} \)

b) \( \frac{3\sqrt{9x}}{x} \)

c) \( \frac{14\sqrt{3} + 2\sqrt{6} + 35 + 5\sqrt{2}}{47} \)
Assessment Example 2:
The time $t(d)$, in seconds, it takes for an object to fall $d$ feet can be modeled by the equation

$$t(d) = \frac{\sqrt{2d}}{g}.$$

Simplify the right-hand side of the model’s equation, i.e., rationalize the denominator.

*Model Assessment Answer(s):*

$$t(d) = \frac{\sqrt{2dg}}{g}.$$

Assessment Example 3:
The distance $d(h)$, in miles, to the horizon at an altitude of $h$ feet above sea level is given by the equation

$$d(h) = \frac{\sqrt{3h}}{2}.$$

Simplify the right-hand side of the model’s equation, i.e., rationalize the denominator.

*Model Assessment Answer(s):*

$$d(h) = \frac{\sqrt{6h}}{2}.$$
COMPETENCY 24

Use multiple representations (tables, graphs, and equations) to solve problems that result in radical equations.

Prior Knowledge:
- Apply properties of radical expressions.
- Add, subtract, and multiply radical expressions.
- Apply properties of exponents.
- Multiply polynomials.

Model Assessments

Assessment Example 1:
Solve:

a) \(2\sqrt{x + 2} = \sqrt{x} + 6\).
b) Using a graph, explain why \(\sqrt{x + 3} = -8\) has no solution.

Model Assessment Answer(s):

a) \(x \approx 31.94\).

b) The range of the graph of the equation \(y = \sqrt{x + 3}\) is greater than 0; therefore, it will never cross.

Assessment Example 2:

Body surface area (BSA) must be calculated for many medical reasons, including renal kidney function, cardiac input and index, and chemotherapy and glucocorticoid dosing. The formula, developed in 1987 by R.D. Mosteller, is:

\[
BSA_{m^2} = \sqrt{\frac{\text{height}_{cm} \cdot \text{weight}_{kg}}{3600}}
\]

Nurse Reynolds has a patient whose BSA has been recorded on the medication chart at 2.04 square meters. If the patient’s height is 157 centimeters (5 ft., 2 in.), what is the patient’s weight in kilograms (rounded to the nearest tenth)?

Model Assessment Answer(s):

\[
2.04_{m^2} = \sqrt{\frac{157_{cm} \cdot \text{weight}_{kg}}{3600}}
\]

Weight = 95.4 kg. (rounded to the nearest tenth).

**COMPETENCY 25**

Apply and graph transformations of parent functions (quadratic, cubic, square root, absolute value, exponential, and logarithmic).

**Prior Knowledge:**
- Recognize the graph of \( y = x^2 \).
- Recognize the graph of \( y = x^3 \).
- Recognize the graph of \( y = \sqrt{x} \).
- Recognize the graph of \( y = |x| \).
- Recognize the graph of \( y = a^x \).
- Recognize the graph of \( y = \ln x \).
- Find the \( x \)- and \( y \)-intercepts of an equation.
- Find the vertex of a parabola.
- Determine if the graph of a parabola opens up or down.
- Apply the properties of logarithms.
- Apply the properties of exponents.

**Model Assessments**

**Assessment Example 1:**
Identify the transformations used to graph the following from their parent graphs:

a) \( f(x) = (x - 3)^2 - 2 \).

b) \( f(x) = -3\sqrt{x + 2} \).

c) \( f(x) = \frac{1}{x + 4} \).

**Model Assessment Answer(s):**

a) \( f(x) = (x - 3)^2 - 2 \); parent graph shifted right 3 units and down 2 units.

b) \( f(x) = -3\sqrt{x + 2} \); parent graph shifted left 2 units and reflected over the \( x \)-axis.

c) \( f(x) = \frac{1}{x + 4} \); parent graph shifted left 4 units.

**Real World Application Reference:**
- Agriculture and Natural Resources
- Engineering and Design
Assessment Example 2:
Solve:

a) \( f(x) = -2\sqrt{x - 5} + 4 \).

b) \( f(x) = 2^{x-1} \).

c) \( f(x) = \log_2(x + 4) \).

Model Assessment Answer(s):

a) \( f(x) = -2\sqrt{x - 5} + 4 \); parent graph shifted right 5 units, vertically stretched by factor of 2, reflected over \( x \)-axis and shifted up 4 units.

b) \( f(x) = 2^{x-1} \); parent graph shifted right 1 unit.

c) \( f(x) = \log_2(x + 4) \); parent graph shifted left 4 units.

Assessment Example 3:
The aerial view of Mrs. Green Thumb's flower garden has one boundary identified as the graph of a parabola and the other boundary is the house itself. The view uses the corner of her house as the origin, with the house sitting in the 3\(^{rd}\) quadrant. The parabola has been reflected with a vertical stretch of \( \frac{1}{4} \) and shifted 5 feet left and 4 feet up. Find the equation of the graph in order to make a sketch of the garden. Use a scale measured in feet.

Model Assessment Answer(s):

The equation is \( y = -\frac{1}{4}(x + 5)^2 + 4 \).
COMPETENCY 26

Identify the parent function (quadratic, cubic, square root, absolute value, exponential, and logarithmic) and transformations for a given transformed function in algebraic or graphical form.

Prior Knowledge:

- Recognize and identify the graph of \( y = x^2 \).
- Recognize and identify the graph of \( y = x^3 \).
- Recognize and identify the graph of \( y = \sqrt{x} \).
- Recognize and identify the graph of \( y = |x| \).
- Recognize and identify the graph of \( y = a^x \).
- Recognize and identify the graph of \( y = \ln x \).
- Graph polynomial functions.
- Identify the \( x \)- and \( y \)-intercepts of a graph of a function.

Model Assessments

Assessment Example 1:

You are designing a header for your company’s stationery. It requires a parabolic arc that begins and ends 1.5 inches below the top edge of the paper, and peaks in the middle of the page 1 inch below the top of the paper. Find an equation for this parabola.

Model Assessment Answer(s):

Transform the parabola \( y = x^2 \), which passes through the points (-1, 1), (0, 0), and (1, 1). If \( x \) and \( y \) are both measured in inches, then this arc has a width of 2 inches and a height of 1 inch. Assuming the page measures 8½ x 11 inches and all distances are measured from the bottom left-hand corner of the page, the target arc has a width of 8½ inches and a height of ½ inch. Stretch the parabola horizontally by a factor of 4.25 and compress it vertically by a factor of 0.5 in order to match the target measurements. Reflect the arc across the \( x \)-axis so that it opens downward, and translate 4.25 inches right and 10 inches up so that its vertex is in the right place. Its equation will be:

\[-(y - 10) = 0.5\left(\frac{x - 4.25}{4.25}\right)^2, \text{ or } y = -0.5\left(\frac{4x}{17} - 1\right)^2 + 10, \text{ or } y = -\frac{8x^2}{289} + \frac{4x}{17} + 9.5.\]

An alternative solution would be to write \( y = ax^2 + bx + c \), use the points (0, 9.5), (4.25, 10), and (8.5, 9.5) to build a 3x3 linear system in \( a \), \( b \), and \( c \), and solve that system.
Assessment Example 2:
A parabola has an equation of \( y = -3x^2 + 18x - 11 \). Complete the square to convert this equation to vertex form; then identify a sequence of transformations that turn the graph of \( y = x^2 \) into the graph of \( y = -3x^2 + 18x - 11 \).

Model Assessment Answer(s):
Complete the square to obtain vertex form equation: \( y = -3(x - 3)^2 + 16 \).

Sequence of transformations:

\[
y = x^2.
\]

Horizontal translation 3 units right (replace variable \( x \) with \( x - 3 \)): \( y = (x - 3)^2 \).

Vertical stretch by factor of 3 (multiply right-hand side by 3 or, equivalently, replace \( y \) with \( \frac{y}{3} \)): \( y = 3(x - 3)^2 \).

Reflection across \( x \)-axis (multiply right-hand side by -1 or, equivalently, replace \( y \) with \( -y \)): \( y = -3(x - 3)^2 \).

Vertical translation 16 units up (add 16 to right-hand side or, equivalently, replace \( y \) with \( y + 16 \)): \( y = -3(x - 3)^2 + 16 \).

Assessment Example 3:
Pair each function in the first column with its parent function in the second column, and identify a sequence of transformations to turn one into the other:

1. \( f(x) = 5(x + 1)^4 - 3 \)  
   A. \( y = \ln x \).
2. \( g(x) = 3e^{x+1} - 7 \)  
   B. \( y = \sqrt{x} \).
3. \( h(x) = \frac{1}{2} \sqrt{4 - x} + 2 \)  
   C. \( y = e^x \).
4. \( p(x) = 2\ln(3x + 2) \)  
   D. \( y = x^4 \).

Model Assessment Answer(s):
1. D. Start with \( y = x^4 \); translate 1 left to get \( y = (x + 1)^4 \); vertically stretch by a factor of 5 to get \( y = 5(x+1)^4 \); translate 3 down to get \( y = 5(x + 1)^4 - 3 \).

2. C. Start with \( y = e^x \); horizontally compress (fixing the \( y \)-axis) by a factor of \( \frac{1}{4} \) to get \( y = e^{4x} \); translate left \( \frac{1}{4} \) to get \( y = e^{4x+1} \); vertically stretch (fixing the \( x \)-axis) by a factor of 3 to get \( y = 3e^{4x+1} \); translate 7 down to get \( y = 3e^{4x+1} - 7 \).

3. B. Start with \( y = \sqrt{x} \); translate 4 left to get \( y = \sqrt{x+4} \); reflect across the \( y \)-axis to get \( y = -\sqrt{x+4} = -\sqrt{4-x} \); compress vertically by a factor of \( \frac{1}{2} \) to get \( y = \frac{1}{2}\sqrt{4-x} \); translate 2 up to get \( y = \frac{1}{2}\sqrt{4-x} + 2 \).

4. A. Start with \( y = \ln x \); horizontally compress (fixing the \( y \)-axis) by factor \( \frac{1}{3} \) to get \( y = \ln(3x) \); translate left \( \frac{1}{3} \) to get \( y = \ln(3(x+2)) \); vertically stretch by a factor of 2 to get \( y = 2\ln(3x + 2) \).
Assessment Example 4:

Name the transformation(s) shown in each picture. Given that the function shown by the solid curve is represented by the equation \( f(x) = x^3 \), find an equation for the function represented by the dashed curve.

(a)

(b)
Algebra II/Intermediate Algebra Competencies

c) 

![Graph c)

-4 -3 -2 -1 1 2 3 4
-4 -3 -2 -1 1 2 3


d) 

![Graph d)

-2 -1 1 2
-3 -2 -1 1 2


e) 

![Graph e)

-3 -2 -1 1 2 3 4 5 6 7 8 9 10
-3 -2 -1 1 2
**Model Assessment Answer(s):**

a) This is a vertical translation, sliding each point 3 units up. The dashed function has the equation \( g(x) = x^3 + 3 \). This is reasonable because \( g(0) = 3 \) and \( g(1) = 4 \).

b) This is a horizontal translation, sliding each point 2 units to the right. The dashed function has the equation \( g(x) = (x - 2)^3 \). This is reasonable because \( g(2) = 0 \) and \( g(3) = 1 \).

c) This is a diagonal translation, sliding each point 1 unit right and 2 units down. (Equivalently, it is the composition of two translations: one of them sliding all points 1 unit to the right and the other sliding all points 2 units down.) The dashed function has the equation \( g(x) = (x - 1)^3 - 2 \). This is reasonable because \( g(1) = -2 \) and \( g(2) = -1 \).

d) This is a horizontal compression by a factor of \( \frac{1}{2} \). The dashed function has the equation \( g(x) = (2x)^3 \). This is reasonable because \( g(0) = 0 \) and \( g\left(\frac{1}{2}\right) = 1 \).

e) This is a vertical stretch by a factor of 8. The dashed function has the equation \( g(x) = 8x^3 \). This is reasonable because \( g(0) = 0 \) and \( g(1) = 8 \). Note that \( (2x)^3 = 8x^3 \), so this figure and the previous one have the same dashed function, even though the transformations and viewing windows are different.
COMPETENCY 27

Compose two linear, quadratic, cubic, square root, absolute value, exponential, or logarithmic functions.

Prior Knowledge:

• Define a function.
• Use function notation.
• Add, subtract, multiply, and divide polynomials to graph functions.
• Recognize the concept of reflection in graphical terms.

Model Assessments

Assessment Example 1:

If \( f(x) = 6x^2 - 2 \) and \( g(x) = \sqrt{x - 5} \),

a) find \( f(g(6)) \).

b) find \( f \circ g \).

Model Assessment Answer(s):

a) \( f(g(6)) = 4 \).

b) \( (f \circ g)(x) = f(g(x)) = 6x - 32 \).

Assessment Example 2:

Let \( h(x) = \frac{2x - 1}{3} \) and \( g(x) = |3x - 7| \). Find \( g(h(4)) \).

Model Assessment Answer(s):

\( g(h(4)) = 0 \).
Assessment Example 3:

An oil tanker struck a submerged rock as the result of a navigation error. The rock ripped a hole in the bottom of the tanker, spilling oil from the oil storage bunkers. The spread of oil leaking from the tanker is in the shape of a circle. The radius of the leakage zone (in feet) after $t$ hours is $r(t) = 200\sqrt{t}$. Find the area $A$ of the leakage zone as a function of time.

Model Assessment Answer(s):

Use $A(r) = \pi r^2$ to represent the area of the circle and given $r(t) = 200\sqrt{t}$.

$$(A \circ r)(t) = A(r(t)) = 40000\pi t.$$
COMPETENCY 28

Find the inverse of linear, quadratic, cubic, square root, absolute value, exponential, and logarithmic functions algebraically and graphically.

Prior Knowledge:
- Use function notation.
- Add, subtract, multiply, and divide polynomials.
- Identify the graphs of the parent functions listed in the competency.
- Apply the concept of a reflection across the line $y = x$.
- Apply properties of exponents.

Model Assessments

Assessment Example 1:
Find the inverse of the function: $g(x) = \frac{2(x - 4)}{5} + 1$.

Model Assessment Answer(s):
$$g^{-1}(x) = \frac{5x + 3}{2}.$$

Assessment Example 2:
Find the inverse of the function: $f(x) = 2(x - 7)^3 - 32$.

Model Assessment Answer(s):
$$f^{-1}(x) = \left( \frac{x + 32}{2} \right)^{\frac{1}{3}} + 7.$$
**Assessment Example 3:**

Given: \( p(x) = \sqrt[3]{\frac{7x - 4}{8}} \), find: \( p^{-1}(x) \).

*Model Assessment Answer(s):*

\[
p^{-1}(x) = \frac{8x^3 + 4}{7}.
\]

**Assessment Example 4:**

The profit made by a company manufacturing shoes is given by \( P(x) = 0.01x^2 + 0.5x - 100 \), where \( x \) represents the number of pairs of shoes produced.

a) Find the inverse of \( P(x) \), \( P^{-1}(x) \) for \( x \geq 0 \).

b) Approximately how many pairs of shoes would the company have to produce to make a profit of $10,000?

*Model Assessment Answer(s):*

a) To find the inverse, let \( y = 0.01x^2 + 0.5x - 100 \).

\[
\begin{align*}
100x &= y^2 + 50y - 10000, \\
100x + 10000 &= y^2 + 50y, \\
100x + 10000 + 625 &= y^2 + 50y + 625, \\
100x + 10625 &= (y + 25)^2, \\
\sqrt{100x + 10625} &= y + 25, \\
y &= \sqrt{100x + 10625} - 25, \\
P^{-1}(x) &= \sqrt{100x + 10625} - 25.
\end{align*}
\]

b) To make a profit of $10,000, find \( P^{-1}(10000) \), which is approximately 981 pairs of shoes.
COMPETENCY 29

Find the equation of an exponential function given two points on the graph.

Prior Knowledge:

• Apply the properties of exponents.
• Use the basic notation of fractional exponents.
• Simplify radicals.

Model Assessments

Assessment Example 1:

Find the equation of an exponential function that passes through the points (2, 264) and (6, 4044) and has a horizontal asymptote at \( y = 12 \).

Model Assessment Answer(s):

\[ y = ab^x + c. \]

Since \( y = 12 \) is an asymptote, \( y = ab^x + 12 \).

Since the graph passes through (2, 264),

\[
264 = ab^2 + 12,
\]

\[
252 = ab^2,
\]

\[
\frac{252}{b^2} = a,
\]

So, \( y = \frac{252}{b^2} b^x + 12 \).

Since the graph passes through (6, 4044),

\[
4044 = \frac{252}{b^2} b^6 + 12,
\]

\[
4032 = 252b^4,
\]

\[
16 = b^4,
\]

\[
2 = b,
\]

\[
\frac{252}{2^2} = a,
\]

\[
63 = a.
\]

So the solution is \( y = (63)2^x + 12 \).
**Assessment Example 2:**

Find the exponential function that passes through (2, 252) and (6, 4032).

*Model Assessment Answer(s):*

\[ y = ab^x. \]

The solution is \( y = (63)2^x \).

**Assessment Example 3:**

If the population of Mathville was 3,020 in 1926 and 5,955 in 1939, determine the population of Mathville in 1984 given steady exponential growth.

*Model Assessment Answer(s):*

\((1926, 3020) \text{ and } (1939, 5955) \rightarrow (0, 3020) \text{ and } (13, 5955)\).

Using \( y = ab^x \), we know that \( a = 3020 \).

\[ \begin{align*}
  y &= ab^x, \\
  5955 &= 3020(b)^{13}, \\
  1.9719... &= b^{13}, \\
  (1.9719...)^{\frac{1}{13}} &= b = 1.054, \\
  y &= 3020(1.054)^x.
\end{align*} \]

The year 1984 is 58 years after 1926, which is why we will use \( x = 58 \).

So, \( y = 3020(1.054)^{58} = 63791 \).

The population in Mathville in 1984 was 63,791.

**Assessment Example 4:**

Five years ago, Sam’s car was worth $15,400. Five years from now it will be worth $4,600. Assuming the depreciation is exponential, what is it worth now? (Round your answer to the nearest dollar).

*Model Assessment Answer(s):*

\[
\begin{array}{c|c}
-5 & 15400 \\
5 & 4600 \\
\end{array}
\]

\[ y = ab^x, \]

\[ b \approx 0.8862, \]

\[ a = 8416, \]

\[ y = 8,416. \]

Sam’s car is currently worth $8,416.
COMPETENCY 30

Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in one- and two-step exponential and logarithmic equations.

Prior Knowledge:

- Recognize inverse functions and their properties.
- Use properties of exponents.
- Use properties of real numbers.
- Apply the properties of exponents.
- Use the basic notation of fractional exponents.
- Simplify radicals.
- Apply the properties of logarithms.
- Apply the properties of exponents.
- Apply the definition/properties of logarithms.
- Apply the laws of exponents.
- Factor numbers into their prime factorizations with appropriate exponentiation.
- Recognize that a single logarithmic expression can be expanded into an equivalent expression.
- Simplify or expand logarithmic expressions.

Real World Application Reference:

- Agriculture and Natural Resources
- Finance and Business
- Health Science and Medical Technology
- Public Services

Model Assessments

Assessment Example 1:

Find the particular exponential equation for the graph shown.
Model Assessment Answer(s):
The equation of this graph is of the form \( y = -c(b)^x \), which passes through the two points \((-1, -2)\) and \((4, -64)\).
The equation of the graph is \( y = -4(2)^x \).

Assessment Example 2:
On an ostrich farm, assume the number of ostriches in a population doubles approximately every 10 months, and the farm begins with 6 ostriches.

a) Write an equation to model the number of ostriches at a given time.
b) How many ostriches will there be after 5 years?
c) When will the number of ostriches reach 10,000?

Model Assessment Answer(s):
a) Equation is: \( P = 6(2)^x \) where \( x \) represents 10-month periods.
b) 5 years = 60 months \( \Rightarrow \) 6 ten-month periods, so \( x = 6 \) and \( P = 384 \) ostriches.
c) \( P = 10,000 \), \( x \approx 10.7 \), so 107 months or 8 years, 11 months.

Assessment Example 3:
A teacher has just contracted a balding bacterial virus. He currently has six bacteria present in his system and the bacteria double every 4.5 hours. When the bacteria reach 1 billion, the teacher will be forced to face baldness.

a) How long does the teacher have before the bacteria hit the critical level?
b) After three days, the teacher shows the first signs of the dreaded disease. He starts taking antibiotics to kill off the virus. The antibiotics will stop the production of any new bacteria and will kill 20% of the virus every hour. When the number of bacteria drops below one, he is cured. When will he be cured?

Model Assessment Answer(s):
a) Find the growth rate of the bacteria by using the exponential function \( P = P_0 e^{kt} \) where \( P \) is the population at a given time, \( t \); \( P_0 \) is the initial population; \( k \) is the growth rate; and \( t \) is the time (in hours). We know that \( P_0 = 6 \) and that when \( t = 4.5 \) hours, \( P \) will be 12. \( t \approx 122.9 \) hours, or about 5 days.
b) The population of the bacteria after three days will be when \( t = 3(24) = 72 \) hours.
\[
P = 6e^{\frac{\ln 2}{6.8}(72)} = 393,216 \text{ bacteria after 3 days.}
\]

A different exponential model is needed to account for the death of 20% of the virus every hour. After 1 hour, the bacteria count will be:
\[
393216 - (0.2)(393216) = (0.8)(393216).
\]

Use this to find the decay rate, \( k \): \( (0.8)(393216) = 393216e^{k(1)} \).
\[
\ln 0.8 = \ln e^k = k \ln e = k.
\]

To find out when he will be cured, let \( P = 1 \) and solve for \( t \):
\[
1 = 383216e^{(\ln 0.8)t}.
\]
\[
t \approx 57.6 \text{ hours, or about 2.4 days.}
\]

**Assessment Example 4:**

The equation \( A = P \left(1 + \frac{0.06}{12}\right)^{12t} \) models the amount of money in a bank account balance after \( t \) years, with interest compounded monthly.

a) Does \( P \) represent the population, interest rate, final balance, starting balance, or time elapsed?

b) What does 0.06 represent?

**Model Assessment Answer(s):**

a) \( P \) represents the starting balance.

b) 0.06 represents the 6% annual interest rate.

**Assessment Example 5:**

The value of Mr. Investor’s stocks is plummeting. Each month his portfolio (the sum of his stocks) loses 4% of its value. Currently the portfolio is worth $512,342.

a) At this rate, what will his portfolio be worth in a year?

b) What was his portfolio worth 8 months ago?
**Model Assessment Answer(s):**

The general form of the equation is

\[ A(t) = A_0 (r)^t, \]

where \( A_0 \) is the initial amount in the portfolio and \( r \) is the percentage rate. Since the stock loses 4\% of its value every month, the percentage rate remaining each period is 96\%. The specific form of the equation for this problem is:

\[ A(t) = 512342 \cdot (0.96)^t, \]

where \( A(t) \) is the amount his portfolio is worth after \( t \) months.

a) To find out what the portfolio will be worth in a year, \( t = 12 \) (since \( t \) is in months) and

\[ A(12) = 512342 \cdot (0.96)^{12} \approx 313,916.94. \]

b) To determine what his portfolio was worth 8 months ago,

\[ A(-8) = 512342 \cdot (0.96)^{-8} \approx 710,215.42. \]

**Assessment Example 6:**

Jennifer had $1,250 in a savings account 3 years ago. The annual interest rate at the time of deposit was 6\%, compounded quarterly.

a) Write an equation to represent the amount of money in the account.

b) How much money is in the account today?

c) How many years will it take for the money in the account to double?

**Model Assessment Answer(s):**

a) \( A(t) = 1250 \cdot (1.015)^t, \) \( t = \) \# of quarters.

b) 3 years = 12 quarters, \( t = 12 \) and \( A \approx 1494.52. \)

c) \( t = 46.56 \) quarters, so the amount will double in 11.6 years.
Assessment Example 7:

a) Write in exponential form: \( \log_2{64} = y \).

b) Write in logarithmic form: \( 9^{2x+1} = 3^{5x} \).

c) Application: Build a table of values to find four points on the graph of \( f(x) = 3^x \).

d) Use this table to graph the function.

e) Use the table to build a corresponding table for \( f^{-1}(x) \).

f) Graph that inverse. Write the domain and range of each.

g) Explain how the domain and range of exponential and logarithmic functions relate to each other.

h) Between which two integers must \( \log_8{21} \) lie?

Model Assessment Answer(s):

a) \( 2^y = 64 \).

b) \( \log_9{3^x} = 2x + 1 \).

c) 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>27</td>
</tr>
</tbody>
</table>

d) Graph \( y = 3^x \) (below); domain \( (-\infty, \infty) \); range \( (0, \infty) \).
f) Graph of the inverse of \( y = 3^x \) (below); domain: \((0, \infty)\); range: \((-\infty, \infty)\).

![Graph of the inverse function](image)

g) For any exponential equation and its inverse logarithmic equation, the domain and range will trade places.

h) Between 1 and 2.

**Assessment Example 8:**

Solve for \( x \):

a) \( 4.2(3.7)^x - 2 = 26.46 \).

b) \( \log_x(5280) = 8 \).

c) \( 18 = 2(3)^{(x+3)} \).

d) \( \log_3(x+3) + \log_3(x-5) = 2 \).

e) \( \log_2(x) - \log_2(x-6) = \log_2(21) \).

f) \( \log_2(10) + \log_2(x+8) = 1 \).
Model Assessment Answer(s):

a) \[
\log \left( \frac{28.46}{4.2} \right) \\
x = \frac{\log 28.46}{\log 4.2}, \\
x \approx 1.4625.
\]

b) \[x \approx 2.9196.\]

c) \[
18 = 2(3^{x+3}), \\
9 = (3^{x+3}), \\
3^2 = (3^{x+3}), \\
2 = x + 3, \\
-1 = x.
\]

d) \[x = 6.\]

e) \[63/10.\]

f) \[-39/5.\]

Assessment Example 9:

Choose the expression that is not equivalent to: \(\log x + \log y - \log z\):

a) \[\log \left( \frac{xy}{z} \right).\]

b) \[\log(x + y + z).\]

c) \[\log xy - \log z.\]

d) \[\log x + \log \left( \frac{y}{z} \right).\]

Model Assessment Answer(s):

b) is not equivalent.
Assessment Example 10:
A scientist left a 32-gram sample of a radioactive substance out and it was exposed to the air. The substance decays at an exponential rate; after 30 minutes, only 26 grams remained.

a) How much of the substance was left when the janitor found it 8 hours after the scientist left?
b) What is the half-life of the substance?

Model Assessment Answer(s):
\[ A(t) = A_0e^{kt}, \]

a) 1.15 grams left after 8 hours.
b) Half-life is about 1.670 hours.

Assessment Example 11:
In a nursing home, the staff must make sure that the cleaning solution will not be harmful to the patients. If the pH is lower than 5, it will be too acidic and could be dangerous. The concentrated solution available has a pH of 4 and needs to be diluted until the pH is 5. The pH scale defines the pH of a solution as 

\[ pH = -\log[H^+] \]

where \([H^+]\) is the concentration of hydrogen ions in the solution in units of moles per liter. If 1 liter of cleaning solution of pH 4 is added to 9 liters of water, what will be the resulting pH of the solution?

Model Assessment Answer(s):
The pH of the starting solution is 4, so we have:

\[ 4 = -\log[H^+] \]

Multiply both sides by -1:

\[ -4 = \log[H^+] \]

then rewrite in exponential form:

\[ 10^{-4} = [H^+] \]

so the concentration of hydrogen ions is \(10^{-4}\) moles/liter.

This needs to be multiplied by the 1 liter of starting solution to give:

\[ (10^{-4} \text{ moles/liter})(1 \text{ liter}) = 10^{-4} \text{ moles}. \]

When 9 liters of water are added, the \(10^{-4}\) moles of hydrogen ions are now in 10 liters of solution, so the concentration is:

\[ \frac{10^{-4} \text{ moles}}{10 \text{ liters}} = 10^{-5} \text{ moles/liter}. \]

To find the resulting pH, 
\[ pH = -\log(10^{-5}) = -( -5) = 5 \]
so the solution will be safe.
Assessment Example 12:
Carbon-14 is used in dating organisms that have died. The half-life for Carbon-14 to decay to Nitrogen-14 is 5,730 years. Find the equation that can be used to determine the amount of Carbon-14, \( C(t) \), remaining after \( t \) years.

Model Assessment Answer(s):
The half-life of a radioactive decay can be determined from the exponential decay equation
\[
C(t) = C_0 e^{-rt}.
\]
The equation is \( C(t) = 100e^{-0.000120968t} \).
COMPETENCY 31

Graph simple conics, including:
• a circle given its equation (centered anywhere)
• an ellipse given its equation (centered at the origin)
• a hyperbola given its equation (centered at the origin)

Prior Knowledge:
• Perform arithmetic computations with rational numbers.
• Graph a linear equation.
• Graph a quadratic equation.
• Convert between general quadratic equation and standard form.
• Interpret the concept of asymptote.
• Draw a rectangle through four given ordered pairs.
• Use the Pythagorean Theorem.
• Use the distance formula.

Real World Application Reference:
• Engineering and Design
• Transportation

Model Assessments

Assessment Example 1:
Graph: $x^2 + (y - 1)^2 = 4$.

Model Assessment Answer(s):
center = $(0, 1)$, radius = 2

![Graph of the circle with center at (0, 1) and radius 2]
Assessment Example 2:

Graph: \( x^2 + y^2 + 4x - 6y + 12 = 0 \).

Model Assessment Answer(s):

\((x + 2)^2 + (y - 3)^2 = 1\).

center = (-2, 3), radius = 1

Assessment Example 3:

There is an explosion in a pipeline in an urban neighborhood. The area of the damage is roughly circular, and the outer limit is described by the equation \( x^2 + y^2 + 2x + 4y - 20 = 0 \), with the point (0, 0) located at the local fire station. Find the location of the explosion (located at the center of the circle) and the blast radius.

Model Assessment Answer(s):

\((x + 1)^2 + (y + 2)^2 = 25\).

The center of the circle (the site of the explosion) is 1 unit west and 2 units south of the fire station. The blast radius is 5 units.
Assessment Example 4:

Find the coordinates of the vertices and foci of the ellipse, then graph the ellipse: \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \).

Model Assessment Answer(s):

center = (0, 0)
vertices = (3, 0) (-3, 0)
foci = \( \pm \sqrt{5}, 0 \)

Assessment Example 5:

An arch in the shape of the upper half of an ellipse is used to support a bridge spanning a river that is 20 meters wide. The center of the arch is 6 meters above the center of the river. Write an equation for the ellipse in which the \( x \)-axis coincides with the water level and the \( y \)-axis passes through the center of the arch. Make a graph to represent this situation.

Model Assessment Answer(s):

The river is 20 meters wide, so \( a = 10 \) meters; the bridge is 6 meters over the water, so \( b = 6 \) meters:

\[
b^2 = a^2 - c^2 \rightarrow 36 = 100 - c^2 \rightarrow c^2 = 64 \rightarrow c = \pm 8.
\]

Equation is: \( \frac{x^2}{10^2} + \frac{y^2}{6^2} = 1 \).
**Assessment Example 6:**

Graph the hyperbola and show the asymptotes:

\[
\frac{x^2}{16} - \frac{y^2}{4} = 1. 
\]

*Model Assessment Answer(s):*

asymptotes: \( y = \frac{1}{2}x \) and \( y = -\frac{1}{2}x \).

![Graph of the hyperbola and asymptotes](image)

**Assessment Example 7:**

The LORAN system (Long Range Navigation) uses hyperbolas to determine a ship's position at sea. Two stations transmit radio pulses to the ship. The difference in the times of arrival of these pulses to the ship is a constant on the hyperbola with the stations as the foci.

Suppose two stations (A to west of center and B to east) are 300 miles apart along a straight section of shore. A ship approaching the shore determines that it is 80 miles farther from station A than it is from station B.

a) Find the equation of the hyperbola on which the ship is located.

b) If the ship is 50 miles offshore, find the coordinates for its location.
Model Assessment Answer(s):

Graph:

![Graph of a hyperbola with stations A and B, and a ship's position.](image)

a) If stations are 300 miles apart, the coordinates of the foci are (-150, 0) and (150, 0) and \( c = 150 \). By definition of a hyperbola, the difference in distances from a point to the foci is equal to \( 2a \), so \( 2a = 80 \) and \( a = 40 \).

For a hyperbola, \( b^2 = c^2 - a^2 \), so \( b^2 = 22500 - 1600 \), or \( 20900 \).

The equation of the hyperbola is: \[ \frac{x^2}{1600} - \frac{y^2}{20900} = 1. \]

b) If the ship is 50 miles from shore, then \( y = 50 \).

\[ \frac{x^2}{1600} - \frac{2500}{20900} = 1, \]

\[ x^2 = 1600 \left(1 + \frac{2500}{20900}\right), \]

\[ x = \pm \sqrt{1791.1388}, \]

\[ x = \pm 42.32. \]

Because the ship is closer to station B, we would use the positive \( x \) value.

The coordinates of the ship are (42.32, 50).
**COMPETENCY 32**

Solve statistical problems:
- Compute permutations using fundamental counting principle.
- Compute combinations.
- Compute probabilities using combinations.
- Compute probabilities using permutations.
- Expand binomial expressions using the binomial theorem.

**Prior Knowledge:**
- Define and use a factorial.
- Apply properties of exponents.

**Model Assessments**

**Assessment Example 1:**
A bank manager assigns passwords to each employee. How many passwords can be assigned using three letters and five digits:

a) with repetitions?
b) without repetitions?

**Model Assessment Answer(s):**
Permutations use the fundamental counting principle. A bank manager can assign:

a) with repetitions, \(3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 16,875\) passwords.

b) without repetitions, \(3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720\) passwords.

**Assessment Example 2:**
A restaurant manager offers a value combo meal with a choice of 12 varieties of sandwiches, 6 varieties of drinks, and 5 varieties of chips. How many possible combo meals are available to customers?

**Model Assessment Answer(s):**
\[12 \cdot 6 \cdot 5 = 360\] possible combo meals.
Assessment Example 3:

a) The team manager of the local coffee shop has to pick four employees to represent their store at the regional sales meeting. How many combinations of employees are possible if there are 15 employees from which to choose?

b) There are three positions open at the local newspaper. If there are 12 applicants, how many staffing assignments are possible?

Model Assessment Answer(s):

Combinations use the fundamental counting principle:

a) \( _{15}C_4 = \binom{15}{4} = 1,365 \) combinations.

b) \( _{12}C_3 = \binom{12}{3} = 220 \) staffing assignments if these three positions are the same, and

\( _{12}P_3 = 12 \times 11 \times 10 = 1,320 \) staffing assignments if these three positions are all different.

Assessment Example 4:

There are 12 people at XYZ Company who want to attend a conference. What is the probability that Jose and Alicia will be the two chosen?

Model Assessment Answer(s):

\( _{12}C_2 = \binom{12}{2} = 66 \) ways to choose 2 people from 12.

\( _2C_2 = \binom{2}{2} = 1 \) way for Jose and Alicia to be chosen together.

The probability is 1/66 = 0.01515.

Assessment Example 5:

A media firm is having a company-wide contest to see who can create the best commercial to market their services. They will be awarding prizes for first, second, and third place winners. There are five departments in the firm and each department has 10 employees. If each employee submits a commercial, what is the probability that the three winning commercials will come from one department?
Model Assessment Answer(s):
There are (5)(10) = 50 total employees.
There are \( \underline{50} P_3 = 117,600 \) ways to award three prizes.
There are \( \underline{8} P_3 = 336 \) ways for a specific department to win all three prizes.
The probability for a specific department to win all three prizes is:
\[
\left( \frac{8}{50} \right) \left( \frac{7}{49} \right) \left( \frac{6}{48} \right) = \frac{336}{117600} = 0.002857.
\]
Any one of the five departments could be the winning department, so the probability that the three winning commercials come from any one department is:
\[
\left( \frac{336}{117600} \right)(5) = 0.01429.
\]

Assessment Example 6:
A dog breeder mates a black lab (b) and a golden retriever (g). The probability of having black fur is 0.6. Write the term in the expansion of \((b + g)^6\) for each of the following outcomes:

a) exactly two black-furred puppies.

b) exactly three golden-furred puppies.

Model Assessment Answer(s):
\[ P(b) = 0.6 \text{ and } P(g) = 1-0.6 = 0.4. \]
In the expansion of \((b + g)^6\) for a total of 6 puppies, the probability of getting

a) exactly two black-furred puppies is:
\[
\binom{6}{2}(P(b))^2(P(g))^4 = 15(0.6)^2(0.4)^4 = 0.1382;
\]

b) exactly three golden-furred puppies is:
\[
\binom{6}{3}(P(b))^3(P(g))^3 = 20(0.6)^3(0.4)^3 = 0.2765.
\]

Assessment Example 7:
Use the binomial theorem to expand the expression \((2x + 3)^5\).

Model Assessment Answer(s):
\[
\binom{5}{0}(2x)^5(3)^0 + \binom{5}{1}(2x)^4(3)^1 + \binom{5}{2}(2x)^3(3)^2 + \binom{5}{3}(2x)^2(3)^3 + \binom{5}{4}(2x)^1(3)^4 + \binom{5}{5}(2x)^0(3)^5
\]
\[
= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243.
\]
COMPETENCY 33

Compute the general term and sums of arithmetic series, finite geometric series, and infinite geometric series.

Prior Knowledge:
- Evaluate algebraic expressions.
- Simplify expressions prior to solving linear equations.
- Solve multistep problems including word problems involving linear and nonlinear equations.
- Use notation involving subscripts and exponents.
- Differentiate between a finite set of numbers vs. an infinite set of numbers.

Model Assessments

Assessment Example 1:
A client of a contractor wants a brick patio floor in the back of his house. He wants 36 bricks to be the first row of his patio floor and each successive row to have two fewer bricks.

a) Write a rule for the number of bricks in the $n$th row.

b) What is the total number of bricks needed for this patio deck, if the last row has 10 bricks?

*Model Assessment Answer(s):*

a) $a_n = 36 - 2(n - 1)$.

b) $S_{14} = 322$ bricks.

Assessment Example 2:
In 2000, about 680 homes were sold in a local community. From 2000 to 2010, the number of homes sold decreased by 7% each year.

a) Write a rule for the total number of homes sold in terms of the year ($n = 1$ for the year 2000).

b) What was the total number of homes sold from 2000 to 2010 (inclusive)?

*Model Assessment Answer(s):*

a) $a_n = 680(0.93)^{n-1}$.

b) $S_{11} = 5,342$ homes.
Assessment Example 3:
An office supply store had a profit of $84,000 the first year it opened. With the depression of today’s economy, their profit has decreased by 15% per year.

a) Write a rule for each year’s profit for the office supply store.
b) With the depression of the economy, what is the total profit the office supply store can make over the course of its lifetime?

Model Assessment Answer(s):

a) \( a_n = 84000(0.85)^{n-1} \).
b) \( S = 560,000 \).
Precalculus Competencies
COMPETENCY 1

Perform matrix operations, including addition and multiplication, and calculate the determinants of 2x2 and 3x3 matrices.

Prior Knowledge:
• Solve systems of equations in two and three variables using substitution and addition/elimination methods.
• Write a system of linear equations in matrix form.
• Perform basic matrix row operations.
• Identify the dimensions of a matrix.
• Identify the elements of a matrix ($a_{11}$, $b_{23}$, etc.).

Model Assessments

Assessment Example 1:
Add the following matrices, if possible. If not possible, explain why not.

Given these matrices,

\[
A = \begin{pmatrix} 6 & 4 & 10 \\ -5 & 2 & -10 \\ -3 & -3 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -5 & -2 \\ 1 & -4 & 7 \\ -9 & 3 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 \\ -5 & 3 \\ 0 & -4 \end{pmatrix},
\]

add the following:

a) $A + C$

b) $B + A$

Model Assessment Answer(s):

a) $A + C$ cannot be performed because the dimensions of $A$ and $C$ are different.

b) $B + A = \begin{pmatrix} 9 & -1 & 8 \\ -4 & -2 & -3 \\ -12 & 0 & -1 \end{pmatrix}$
Assessment Example 2:

Apply matrix operations to solve problems involving multiplication. Multiply the following matrices, if possible. If not possible, explain why not.

\[
\begin{pmatrix}
-3 & 2 \\
5 & 1 \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{2} & 4 & -5 \\
3 & \frac{2}{3} & -6 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
\frac{1}{2} & 4 & -5 \\
3 & \frac{2}{3} & -6 \\
\end{pmatrix}
\begin{pmatrix}
-3 & 2 \\
5 & 1 \\
\end{pmatrix}
\]

Model Assessment Answer(s):

a) \[
\begin{pmatrix}
\frac{9}{2} & \frac{-32}{3} & 3 \\
11 & \frac{62}{3} & -31 \\
\end{pmatrix}
\]

b) Matrices cannot be multiplied because the inner dimensions do not match.

Assessment Example 3:

A contractor is developing a budget for her current construction projects. Her company is building single-family homes, duplexes, and condominium complexes. The costs for the various categories of building supplies are as follows: lumber costs $5,000 for a single-family home, $8,000 for a duplex, and $125,000 for a condominium complex. The costs for windows are $3,000 for a single-family home, $5,000 for a duplex, and $75,000 for a condominium complex. Concrete costs $7,500 for a single-family home, $12,000 for a duplex, and $170,000 for a condominium complex. Plumbing supplies for each run $2,500, $3,500, and $50,000 for the single-family home, duplex, and condominium complex, respectively. Roofing materials for each run $5,000, $8,000, and $100,000 for the single-family home, duplex, and condominium complex, respectively.

This year she is planning to build 10 single-family homes, 4 duplexes, and 7 condominium complexes. Use matrices to determine her total costs this year for each category of construction materials.

Model Assessment Answer(s):

\[
\begin{bmatrix}
5000 & 8000 & 125000 \\
3000 & 5000 & 75000 \\
7500 & 12000 & 170000 \\
2500 & 3500 & 50000 \\
5000 & 8000 & 100000 \\
\end{bmatrix}
\begin{bmatrix}
10 \\
4 \\
7 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Lumber} & \text{Windows} & \text{Concrete} & \text{Plumbing} & \text{Roofing} \\
957000 & 575000 & 1313000 & 389000 & 782000 \\
\end{bmatrix}
\]
Assessment Example 4:
Apply matrix operations to solve problems involving determinants. Find the determinant of the following matrices, if they exist:

a) \[
\begin{pmatrix}
-3 & 2 \\
5 & 1
\end{pmatrix}
\]

b) \[
\begin{pmatrix}
\frac{1}{2} & 4 & -5 \\
3 & \frac{2}{3} & -6
\end{pmatrix}
\]

c) \[
\begin{pmatrix}
4 & 2 & -1 \\
2 & 1 & -2 \\
-5 & 10 & 0.5
\end{pmatrix}
\]

Model Assessment Answer(s):

a) -13.
b) Determinant does not exist; not a square matrix.
c) 75.

Assessment Example 5:
Clarisse invested a total of $60,000 in money market accounts at three different banks. Bank A pays 2% interest per year, Bank B pays 2.5%, and Bank C pays 3%. She decided to invest twice as much in Bank B as in the other two banks combined. After 1 year, Clarisse had earned $1,575 in interest. How much did she invest in each bank? Use Cramer’s Rule to solve this system of equations.

Model Assessment Answer(s):

\[A + B + C = 60,000,\]
\[2A - B + 2C = 0,\]
\[0.02A + 0.025B + 0.03C = 1575.\]

Bank A = $2,500
Bank B = $40,000
Bank C = $17,500
COMPETENCY 2

Describe different types of functions (polynomial, exponential, logarithmic, and trigonometric) using a table, a graph, an equation, and a verbal description.

Prior Knowledge:

- Determine the zeros (x-intercepts) of polynomial functions.
- Factor polynomials using standard factoring techniques.
- Apply appropriate function vocabulary (domain, range, asymptotes, etc.) to describe properties of functions.

Model Assessments

Assessment Example 1:
ACCESS Food Service is preparing pies to serve for dessert in the dining commons. They want to serve the pies as soon as possible, but need them to cool after removing them from the oven. To avoid injuries, the pies need to have an internal temperature of 120°F or lower. The rate of cooling will follow a decreasing exponential function that approaches the ambient temperature of the room, which is 75°F. A pie is taken out of the oven and its temperature is measured.

- After 2 minutes, the pie's temperature is 311°F.
- After 5 minutes, the pie's temperature has dropped to 276°F.

a) Sketch a graph that shows how each pie's temperature decreases over time after being taken out of the oven. Include all important features of the graph.

b) Temperature can be modeled by the function $T(t) = am^{-t} + k$. The constant $a$ is the temperature difference between the pie and the ambient room temperature at $t = 0$ and $m$ is the temperature decay rate. What is the value of $k$?

c) Using the data from above, find the function for the pie temperature.

d) What temperature is the pie when it first comes out of the oven (time = 0)?

e) When is it safe to serve the pies?
Model Assessment Answer(s):

![Graph](image)

a) The graph starts at the y-intercept of the temperature at which it is removed from the oven. The horizontal asymptote is at the room temperature, which is 75°F.

b) The function is \( T(t) = am^{-t} + k \), so the value of \( k \) is room temperature, which is 75°F.

c) The equation is \( T(t) = (262.7)(1.055)^{-t} + 75 \).

d) \( T(0) = 337.7 \); the pie comes out of the oven at 337.7°F.

e) To find when the pie can be served, set the temperature to 120°F and solve for \( t \).

   The pie can be served after 33 minutes.

Assessment Example 2:

A cabinet maker needs to make a rectangular drawer that would have a volume of 4,500 cubic centimeters. Because of what the drawer will hold, the length of the drawer needs to be twice the width.

a) Determine the formula for the surface area as a function of the width of the drawer.

b) Make a table of the surface area as a function of the width of the drawer.

c) Draw a graph of the surface area as a function of the width of the drawer.

d) Use a graphing calculator to find the height so that the surface area is as small as possible to use the least amount of wood.
Model Assessment Answer(s):

Let \( L = \) length of the drawer, \( W = \) width of the drawer, \( H = \) height of the drawer, and \( V = \) volume of the drawer.

\[ V = LWH = 4,500 \text{ cubic centimeters.} \]

a) Surface area = \( SA(W) = 2W^2 + 13500 / W \).

b) Table:

<table>
<thead>
<tr>
<th>Width (W)</th>
<th>Surface Area = 2W^2 + 13,500/W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2,750</td>
</tr>
<tr>
<td>10</td>
<td>1,550</td>
</tr>
<tr>
<td>15</td>
<td>1,350</td>
</tr>
<tr>
<td>20</td>
<td>1,475</td>
</tr>
<tr>
<td>25</td>
<td>1,790</td>
</tr>
</tbody>
</table>

c) Graph

The graph shows that the minimum surface area appears to occur when \( W = 15 \). When \( W = 15 \) cm., \( H = 10 \) cm.
Assessment Example 3:

If the cost, in dollars, of cleaning up an oil spill is given by the function \( f(x) = \frac{2,000,000}{100 - x} \), where \( x \) is the percent of the spill that has been cleaned up:

a) How much will it cost to clean up 50% of the spill? 80%? 90%?

b) Draw a graph of the cost function. Describe the behavior of the graph.

c) If the company is going to spend $4,000,000 cleaning up the spill, what percent will be cleaned up?

d) Will 100% of the spill be cleaned up? What feature of the graph leads you to this conclusion?

Model Assessment Answer(s):

a) Find the values of the function where \( x = 50 \), then 80, then 90.

\[
\begin{align*}
\text{f}(50) &= 40000, \\
\text{f}(80) &= 100000, \\
\text{f}(90) &= 200000.
\end{align*}
\]

It will cost $40,000 to clean up 50% of the spill, $100,000 to clean up 80%, and $200,000 to clean up 90%.

b) Graph

![Graph of cost function]

\[
\begin{align*}
\text{Cost of cleanup (}) \\
\text{Percent of spill (%)} \\
\end{align*}
\]

c) Set \( f(x) = 4,000,000 \) and solve for \( x \).

So if you spend $4,000,000, you will be able to clean up 99.5% of the spill.

d) It’s impossible to clean up the entire spill. The amount of money approaches infinity as \( x \) approaches 100%.
Assessment Example 4:

As sand is being unloaded from a hopper at a construction site, the rising pile is in the shape of a cone. As more and more sand pours onto the pile, the pile grows but the cones at different stages are always similar. After 1 minute, the height of the cone is 5 feet and the radius of the base is 9 feet. After 3 minutes, the height is 12 feet.

a) Find an equation for the area of the base as a function of time.

b) How much area on the ground does the pile cover after 3 minutes?

Model Assessment Answer(s):

a) Find the area of the base as a function of time.

Volume formula for a right circular cone: \( V = \frac{1}{3}\pi r^2 h \), where \( r \) = radius of the circular base and \( h \) = height of the cone. Assuming that the sand is pouring out at a constant rate, the radius of the base will change at a linear rate.

Area of a circle: \( A = \pi r^2 \), where \( r = 6.3t + 2.7 \) and \( t \) is time.

\[ A(t) = \pi (6.3t + 2.7)^2 \]

b) Area covered at \( t = 3 \).

\[ A(3) = \pi (6.3(3) + 2.7)^2 = \pi (21.6)^2 \text{ sq. ft.} \]
COMPETENCY 3

Graph piecewise-defined functions.

Prior Knowledge:

- Graph linear, quadratic, and radical functions.
- Identify the domain and range of a function.

Model Assessments

Assessment Example 1:

a) Graph: \( f(x) = \begin{cases} \frac{1}{4}x^2 & \text{for } -2 < x \leq 2, \\ \sqrt{x + 2} & \text{for } x > 2. \end{cases} \)

b) Graph: \( f(x) = \begin{cases} -x & \text{if } x < 0, \\ x^2 - 1 & \text{if } 0 \leq x < 2, \\ 1 & \text{if } x > 2. \end{cases} \)

Model Assessment Answer(s):

a) [Graph image]
Assessment Example 2:

Honest Abe Finance had to revalue their portfolio in mid-2008. Their monthly profit for month \( t \) of 2008 is given by:

\[
P(t) = \begin{cases} 
25000t + 500000, & 0 < t < 6, \\
75000 - 50000t, & t \geq 6.
\end{cases}
\]

Graph this function for 2008.

Model Assessment Answer(s):
**Assessment Example 3:**

San Calpassia Gas & Electric charges its residential customers a base rate of $7.00 per month plus $0.06 per kilowatt-hour (kWh) for the first 491 kWh, and $0.111 per kWh for all usage above that amount. There is an additional surcharge of $0.02 per kWh for electric usage during peak-demand hours.

a) Graph the energy charge function (amount customers have to pay) for normal usage, not including the peak-demand surcharge, up to 1,000 kWh of electricity usage.

b) During the month of August 2008, your energy usage was 753 kWh. Of that 753 kWh, 75 kWh were used during the peak-demand time period. Determine the total cost of the energy bill for the month of August 2008, excluding taxes and additional surcharges.

**Model Assessment Answer(s):**

\[ C(x) = \begin{cases} 
7.00 + 0.06x & \text{for } x \leq 491 \text{ kWh}, \\
36.46 + 0.111(x - 491) & \text{for } x > 491 \text{ kWh}. 
\end{cases} \]

a) Energy charge function for San Calpassia Gas & Electric:

![Graph showing energy charge function](image)

b) \[ C(753) + 0.02(75) = 67.04. \]
COMPETENCY 4

Find all real zeros (roots) of polynomial functions exactly using the rational root theorem.

Prior Knowledge:

- Apply standard factoring techniques to polynomials.
- Divide polynomials using long division and synthetic division.
- Solve polynomial equations.

Model Assessments

Assessment Example 1:
List the possible rational zeros; find all real zeros:

\[ f(x) = x^3 + 4x^2 - x - 10. \]

Model Assessment Answer(s):

Possible rational zeros: \( \pm 1, \pm 2, \pm 5, \pm 10. \)

Real zeros: \(-2, -\frac{1}{2} \pm \sqrt{6}.\)

Assessment Example 2:
List the possible rational zeros; find all real zeros:

\[ f(x) = x^3 - 3x^2 + 3x - 9. \]

Model Assessment Answer(s):

Possible rational zeros: \( \pm 1, \pm 3, \pm 9. \)

Real zeros: 3.

Real World Application Reference:

- Manufacturing and Product Development
Assessment Example 3:
In metal shop, you have been tasked with creating a prototype metal tray that is open on top. Your starting material is a rectangular sheet of tin that is 22 centimeters by 28 centimeters. You are to cut squares of equal sizes from each corner and fold the resulting flaps up on each side to form the tray.

a) Find a formula for the volume \( V(x) \).
b) What is the domain of this function?
c) Sketch the graph of \( V(x) \).
d) For which value(s) of \( x \) does the tray have a volume of 1,080 squared centimeters?

Model Assessment Answer(s):

a) \( V(x) = x(28 - 2x)(22 - 2x) = 4x^3 - 100x^2 + 616x \).
b) \( x \in (0,11) \).
c) Graph of \( V(x) \):

d) Two solutions: \( x = 5 \) cm. or \( x \approx 3.2 \) cm.
COMPETENCY 5

Find all the zeros (roots) of a polynomial function.

Prior Knowledge:
• Divide polynomials using long division and synthetic division.
• Apply standard factoring techniques to polynomials.
• Solve polynomial equations.
• Identify real and complex solutions to polynomial equations.

Model Assessments

Assessment Example 1:
Find all the zeros (roots) of the functions below.

a) \( f(x) = x^3 - 3x^2 + 3x - 9 \).
b) \( g(x) = x^4 - 2x^3 - 2x^2 - 2x - 3 \).
c) \( h(x) = x^5 - 2x^4 + 2x^3 - 4x^2 + x - 2 \).

Model Assessment Answer(s):

a) \( f(x) \) are \( x = 3, \pm i\sqrt{3} \).
b) \( g(x) \) are \( x = -1, 3, \pm i \).
c) \( h(x) \) are \( x = 2, + i \) (multiplicity of 2), \(-i\) (multiplicity of 2).
**Assessment Example 2:**

You have been tasked to design a box for shipping a new product. The box is to be made from a single sheet of cardboard that is 30 centimeters by 60 centimeters. The box is to be formed by cutting out equal size square pieces from each corner and folding up the sides. The resulting box formed has to have a volume of 2,400 cubic centimeters. What are the dimensions of the cutout sections of the cardboard to the nearest centimeter?

*Model Assessment Answer(s):*

There are two solutions to the problem: \( V(x) = x(60 - 2x)(30 - 2x) \) where \( x \) is the dimension of a side of the cutout square.

- Solution 1: 1.57 cm.
- Solution 2: 12.24 cm.

**Assessment Example 3:**

You and your friends have decided that all the current energy drinks on the market are lacking in a special ingredient that you have found makes a significant difference in terms of performance and lack of “crashing” as the effects wear off. You have done some preliminary marketing analysis and determined that the demand function \( D(x) \), which relates the number of cases consumers will purchase at a specific price \( x \), is given by the following polynomial:

\[
D(x) = x^2 - 200x + 10000 \quad \text{for} \quad 0 \leq x \leq 100.
\]

The supply function \( S(x) \) gives the number of cases you can make if you are selling them at a price of \( x \) dollars and is given by the following:

\[
S(x) = x^3 + 3x^2 - 500x - 5000 \quad \text{for} \quad 0 \leq x \leq 100.
\]

The business experts say that you should set the price of your product so that the supply equals demand, i.e., \( S(x) = D(x) \).

a) What price do you have to charge just to make the first case? (HINT: \( S(x) = 0 \).)
b) At that price, how many cases do you expect to sell?
c) What price should you charge for the business experts to be happy?

*Model Assessment Answer(s):*

a) \$8.39.
b) 8,392 cases.
c) \$15.46.
**COMPETENCY 6**

Solve polynomial inequalities.

**Prior Knowledge:**
- Solve linear inequalities and graph the solution space.
- Solve polynomial equations.

**Model Assessments**

**Assessment Example 1:**
Solve for $x$: $x^3 - 5x^2 + 6x < 0$.

*Model Assessment Answer(s):*

$x(x^2 - 5x + 6) < 0,$

$x(x - 3)(x - 2) < 0.$

This will be true for $x < 0$ or $2 < x < 3$ or $(-\infty, 0) \cup (2, 3)$ in interval notation.

**Assessment Example 2:**
Solve for $x$: $x^3 - 2x^2 - 3x > 0$.

*Model Assessment Answer(s):*

$x(x^2 - 2x - 3) \geq 0,$

$x(x - 3)(x + 1) \geq 0.$

This will be true for $-1 \leq x \leq 0$ and for $x \geq 3$ or $[-1, 0] \cup [3, \infty)$ in interval notation.

**Real World Application Reference:**
- Building Trades and Construction
Assessment Example 3:
A rectangular enclosure for storing construction equipment must have an area of at least 3,200 square yards. If there are 240 yards of fencing available, and the width cannot exceed the length, within what limits must the width of the enclosure lie?

Model Assessment Answer(s):
The width must be between 40 and 60 yards, inclusive.
COMPETENCY 7

Solve rational inequalities.

Prior Knowledge:
• Solve linear inequalities.
• Solve rational equations.

Model Assessments

Assessment Example 1:
Solve for $x$: $\frac{x + 4}{x + 2} > 0$.

Model Assessment Answer(s):
$(−∞, −4)∪(−2, ∞)$.

Assessment Example 2:
Solve for $x$: $\frac{5}{x - 1} + \frac{4}{x} \geq -2$.

Model Assessment Answer(s):
$\frac{5}{x - 1} + \frac{4}{x} + 2 \geq 0$ simplifies to $\frac{(2x - 1)(x + 4)}{x(x - 1)} \geq 0$.

Critical numbers are: $\frac{1}{2}, -4, 0, 1$.
The solution set is $(−∞, −4]∪(0, \frac{1}{2}]∪(1, ∞)$. 

Real World Application Reference:
• Finance and Business
• Manufacturing and Product Development
• Marketing, Sales, and Service
Assessment Example 3:
The set-up cost for manufacturing a roller for a large printer is $2,000. The cost to manufacture each roller is $1,050. Write a formula for the cost, \( C(x) \), as a function of the number of rollers produced. The average cost is given by \( \frac{C(x)}{x} \). If the customer is willing to pay no more than $1,200 for each roller, what would the minimum order need to be?

Model Assessment Answer(s):
Solution:
\[
C(x) = 2000 + 1050x,
\]
\[
x > \frac{40}{3}.
\]

However, because the answer must be a natural number (the question is about discrete objects), the minimum order would be 14 rollers.
COMPETENCY 8

Find the composition of two or more functions, each containing two or more operations (such as linear, quadratic, rational, cubic, square root, cube root, and trigonometric functions).

Prior Knowledge:

- Evaluate functions.
- Identify the domain and range of functions.
- Identify an algebraic expression as either a function or a relation.

Model Assessments

Assessment Example 1:

a) Let \( f(x) = 4x^2 + 6 \) and \( g(x) = 3x - 7 \). Find \( f(g(x)) \) and \( f(f(x)) \).

b) Let \( f(x) = \frac{3}{x} \) and \( g(x) = \frac{x+4}{x-5} \). Find \( f(g(x)) \).

c) Let \( h(x) = \sqrt{x^2 - 4} \). Find two functions \( f \) and \( g \) such that \( h(x) = f(g(x)) \).

Model Assessment Answer(s):

a) \( f(g(x)) = f(3x - 7) = 4(3x - 7)^2 + 6 = 36x^2 - 168x + 202 \),
\( f(f(x)) = f(4x^2 + 6) = 4(4x^2 + 6)^2 + 6 = 64x^4 + 192x^2 + 150 \).

b) \( f(g(x)) = f\left(\frac{x+4}{x-5}\right) = \frac{3(x-5)}{x+4} \) or \( \frac{3x-15}{x+4} \).

c) Let \( g \) be the “inside” function: \( g(x) = x^2 - 4 \).
Let \( f \) be the “outside” function: \( f(x) = \sqrt{x} \).
Assessment Example 2:

A floor refinishing company charges per square foot to refinish a hardwood floor. The area of a certain square floor is given by $A(s)$, where $s$ is the length of a side of the floor. A gallon of varnish costs $45 and will cover 300 square feet of hardwood floor.

a) Express the cost $C(A)$ as a function of $s$.

b) Find the cost of refinishing a room with length 25 feet.

*Model Assessment Answer(s)*:

a) The area of a square floor is $A(s) = s^2$, where $s$ is the length of a side of the floor.

The varnish costs $45 per 300 square feet, so $C(A) = 45 \left( \frac{A}{300} \right)$.

So $C(A(s)) = C(s^2) = 45 \left( \frac{s^2}{300} \right) = C(s)$.

b) $C(25) = 45 \left( \frac{25^2}{300} \right) = 93.75$,

or, assuming that the company charges by the gallon of varnish used:

$A(25) = 25^2 = 625$.

$625 \div 300 = 2 \frac{1}{12}$, so 3 gallons of varnish are needed.

$45(3) = 135$. 
**COMPETENCY 9**

Given a function, determine if an inverse function exists. If so:

- Find the inverse of the function algebraically and graphically.
- Identify the domain and range of the original and inverse functions.

**Prior Knowledge:**

- Identify if a relation is a function.
- Graph linear, polynomial, and radical functions.
- Identify the domain and range of functions and relations.
- Apply transformations to the graphs of functions, including reflection.

**Model Assessments**

**Assessment Example 1:**

Given the following functions, determine if the inverse function exists. If so, find the inverse function and identify the domain and range of the original and inverse functions.

a) \( y = 2^{x+1} + 4 \).

b) \( g(x) = 3\sin 2x \).

c) \( h(x) = \frac{1}{2}\tan^{-1} x \).

d) \( f(x) = 2\sqrt{x+3} \).

Sketch each function and its inverse:

e) \( f(x) = \frac{x-5}{x+2} \).

f) \( h(x) = 4^x \).
Model Assessment Answer(s):

a) For $x > 4$, $f^{-1}(x) = \log_2(x - 4) - 1$,
   
domain of $f(x) = \{x \mid x \in \mathbb{R}\}$, range of $f(x) = \{y \mid y > 4\}$;
   
domain of $f^{-1}(x) = \{x \mid x > 4\}$, range of $f^{-1}(x) = \{y \mid y \in \mathbb{R}\}$.

b) $g^{-1}(x) = \frac{1}{2} \sin^{-1}\left(\frac{x}{3}\right)$,
   
domain of $g(x) = \left\{x \mid \frac{\pi}{4} \leq x \leq \frac{\pi}{4}\right\}$, range of $g(x) = \left\{y \mid -3 \leq y \leq 3\right\}$;
   
domain of $g^{-1}(x) = \left\{x \mid -3 \leq x \leq 3\right\}$, range of $g^{-1}(x) = \left\{y \mid \frac{\pi}{4} \leq y \leq \frac{\pi}{4}\right\}$.

c) $h^{-1}(x) = \tan 2x$,
   
domain of $h(x) = \{x \mid x \in \mathbb{R}\}$, range of $h(x) = \left\{y \mid -\frac{\pi}{4} < y < \frac{\pi}{4}\right\}$;
   
domain of $h^{-1}(x) = \left\{x \mid -\frac{\pi}{4} < x < \frac{\pi}{4}\right\}$, range of $h^{-1}(x) = \{y \mid y \in \mathbb{R}\}$.

d) $f^{-1}(x) = \left(\frac{x}{2}\right)^2 - 3$,
   
domain of $f(x) = \{x \mid x \geq -3\}$, range of $f(x) = \{y \mid y \geq 0\}$;
   
domain of $f^{-1}(x) = \{x \mid x \geq 0\}$, range of $f^{-1}(x) = \{y \mid y \geq -3\}$. 
e) Graph of \( y = \frac{x - 5}{x + 2} \)

Find the inverse of the function: \( y = \frac{x - 5}{x + 2} \)

\[ f^{-1}(x) = \frac{2x + 5}{1 - x}. \]

f) Graph of \( h(x) = 4^x \) and \( h^{-1}(x) = \log_4 x = \frac{\log x}{\log 4} \).
Assessment Example 2:
You have been asked to provide the ground beef for a barbecue at a local festival. You need a total of 50 pounds of two types of ground beef costing $1.25 and $1.60 per pound, respectively. A model for the total cost $C(x)$ of the two types of beef is:

$$C(x) = 1.25x + 1.60(50 - x), \ 0 \leq x \leq 50,$$

where $x$ is the number of pounds of the less expensive ground beef.

a) Find the inverse of the cost function. What does each variable represent in the inverse function?

b) Use the context of the problem to determine the domain of the inverse function.

Model Assessment Answer(s):

a) $C^{-1}(x) = \frac{-20x + 1600}{7}$. In this function, $x$ is the total cost. If you know the total cost, you can calculate how many pounds of the less expensive ground beef you need.

b) The domain of the inverse function would be $62.50 \leq x \leq 80$ (dollars).
COMPETENCY 10

Simplify expressions involving exponents. (This is a prerequisite skill that is used in Calculus.)

Prior Knowledge:
• Apply the properties of exponents.
• Convert between rational exponents and their corresponding radical forms.

Model Assessments

Assessment Example 1:
Simplify:

a) \( \frac{5a^2b^5c^{-6}}{a^{-5}b^{20}c^{15}} \).

b) \( \left[ 32^{\frac{3}{4}} \right]^{\frac{2}{5}} \).

c) \( \frac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}} \).

d) \( [ (3x - 2)^{7m+5} ]^5 (3x - 2)^{2n+9} \).

Model Assessment Answer(s):

a) \( \frac{5a^8}{b^{15}c^{21}} \).

b) \( 2^{6/7} \).

c) \( \frac{xy}{y - x} \).

d) \( (3x - 2)^{35m+2n+34} \).
**COMPETENCY 11**

*Expand logarithmic expressions. (This is a prerequisite skill that is used in Calculus.)*

**Prior Knowledge:**
- Apply the laws of exponents.
- Expand simple logarithmic expressions into equivalent expressions.
- Convert expressions between rational exponents and their corresponding radical forms.

**Model Assessments**

**Assessment Example 1:**
Write the following expression as a sum and/or difference of logarithms or multiples of logarithms:

\[
\ln \left( \frac{\sqrt[3]{2x + 3} \left( x^2 + 3 \right)^5}{\sqrt[5]{x + 2} (3x + 5)^8} \right).
\]

*Model Assessment Answer(s):*
\[
\frac{1}{3} \ln(2x + 3) + 5 \ln(x^2 + 3) - \frac{1}{2} \ln(x + 2) - 8 \ln(3x + 5).
\]

**Assessment Example 2:**
Write the expression as a sum and/or difference of logarithms or multiples of logarithms:

\[
\log_5 \sqrt[3]{\frac{x^2 y}{25}}.
\]

*Model Assessment Answer(s):*
\[
\frac{2}{3} \log_5 x + \frac{1}{3} \log_5 y - \frac{2}{3}.
\]
Assessment Example 3:
Write the expression as a sum and/or difference of logarithms or multiples of logarithms:
\[ \log \left[ x^{\frac{1}{2}} (x - 1)^4 \right]. \]

Model Assessment Answer(s):
\[ \frac{1}{2} \log x + 4 \log (x - 1). \]
COMPETENCY 12

Condense logarithmic expressions. (This is a prerequisite skill that is used in Calculus.)

Prior Knowledge:
• Apply the laws of exponents.
• Expand simple logarithmic expressions into equivalent expressions.
• Convert expressions between rational exponents and their corresponding radical forms.

Model Assessments

Assessment Example 1:
Write the following expressions as a single logarithmic expression:

a) \( \log \sqrt{x-1} - \log \left( \frac{x^2-1}{x+1} \right) \).

b) \( 3[3\ln x + \ln y] + 2[\ln y + 2\ln z] \).

c) \( 4\log_b x - 2\log_b 6 - \frac{1}{2}\log_b y \).

Model Assessment Answer(s):

a) \( \log \left( \frac{1}{\sqrt{x-1}} \right) \).

b) \( \ln \left( x^9 y^5 z^4 \right) \).

c) \( \log_b \left( \frac{x^4}{36\sqrt{y}} \right) \).
COMPETENCY 13

Solve multistep exponential equations.

Prior Knowledge:

- Apply properties of exponents in simplifying expressions.
- Apply properties of logarithms in simplifying expressions.
- Convert between exponential and logarithmic forms of an equation.

Model Assessments

Assessment Example 1:

Solve the following exactly: $e^{2x+1} = 5^x$.

*Model Assessment Answer(s):*

$$x = \frac{1}{\ln(5)-2}.$$

Assessment Example 2:

At 7% interest, compounded annually, how many years would it take an initial investment of $6,000 to reach $11,802.91? (Round to the nearest year.)

*Model Assessment Answer(s):*

$$x = \frac{\ln\left(\frac{11802.91}{6000}\right)}{\ln(1.07)} \approx 10 \text{ years}.$$
Assessment Example 3:

You have been given a bonus at work and you want to invest it in a CD or money market fund. You have two options: a money market fund that pays 7% interest, compounded monthly, and a CD that pays 5%, compounded continuously. In order to evaluate the effectiveness of each option, you want to compare the doubling time for each. Calculate the doubling time of each investment (round to the nearest year):

a) $14,500 at 7% interest, compounded monthly.
b) $14,500 at 5% interest, compounded continuously.

Model Assessment Answer(s):

a) \[29000 = 14500 \left(1 + \frac{0.07}{12}\right)^{12x},\]

\[\frac{\ln\left(\frac{29000}{14500}\right)}{12\ln\left(1 + \frac{0.07}{12}\right)} = x \approx 10 \text{ years}.\]

b) \[29000 = 14500e^{0.05x},\]

\[\frac{\ln\left(\frac{29000}{14500}\right)}{0.05} = x \approx 14 \text{ years}.\]

Assessment Example 4:

You are conducting research into the decline of a very large forested area that consists of about 2.6 million trees. Some kind of disease is attacking the trees causing them to die at the rate of 3% per year. How many years from now will the forest have shrunk to 1.5 million trees?

Model Assessment Answer(s):

\[1.5 = 2.6(1-0.03)^x,\]

\[\frac{\ln\left(\frac{1.5}{2.6}\right)}{\ln(0.97)} = x \approx 18 \text{ years from now}.\]
COMPETENCY 14

Solve multistep logarithmic equations.

Prior Knowledge:

- Convert exponential equations to logarithmic equations and vice versa.
- Apply the properties of logarithms in simplifying expressions and equations.
- Apply the properties of exponents in simplifying expressions and equations.

Model Assessments

Assessment Example 1:
Solve:

a) \( \log_2(x + 2) + \log_2(x - 5) = 3 \).

b) \( \log_5(x - 1) + \log_5(x + 1) = \log_5(x + 5) \).

Model Assessment Answer(s):

a) Solution: \( x = 6 \).

b) Solution: \( x = 3 \).

Assessment Example 2:
As part of a research team investigating the effects of a new drug, Calpassia, you are charged with creating a mathematical model that describes how much of the drug remains in the body at any given time. It is known that 15% of Calpassia is metabolized (used up) by the body each hour. If a dose of 45 milligrams is given at time \( t = 0 \), write an exponential function that gives the amount remaining after \( t \) hours. At what time will 12 milligrams remain? Round your answer to the nearest hundredth.

Model Assessment Answer(s):

The general form of the function is \( A(t) = A_0 e^{kt} \), where \( A(t) \) is the amount remaining after \( t \) hours, \( A_0 \) is the initial amount, \( k \) is the decay constant, and \( t \) is time in hours. For this particular problem, the function is \( A(t) = 45 e^{(\ln 0.85)t} = 45 e^{-0.1625t} \).

At \( t = 8.13 \) hours, 12 milligrams of Calpassia will remain in the body.
COMPETENCY 15

Sketch the graph of a conic, identifying center, vertices, foci, and asymptotes (if present).

Prior Knowledge:
- Graph linear, polynomial, and radical equations.
- Use completing the square to solve quadratic equations.

Model Assessments

Assessment Example 1:
Sketch the graph of $4(x + 1)^2 - 9(y - 5)^2 = 36$ and identify the vertices, foci, and asymptotes.

Model Assessment Answer(s):
Equation in standard form is: \[ \frac{(x + 1)^2}{9} - \frac{(y - 5)^2}{4} = 1 \], hyperbola, center at (-1, 5).

Vertices are: (-4, 5) and (2, 5); foci are: $(-1 - \sqrt{13}, 5)$ and $(-1 + \sqrt{13}, 5)$; asymptotes are:
\[ y = \frac{2}{3} x + \frac{17}{3}, \quad y = -\frac{2}{3} x + \frac{13}{3}. \]
Assessment Example 2:

A comet is due to return to Earth space in the near future. You are tasked with presenting a description of the comet’s path. The path of the comet in its elliptical orbit around the sun can be modeled by the equation: \(9x^2 + 16y^2 = 144\).

a) Sketch the graph of the comet’s orbit.

b) The sun is located at one of the foci of the ellipse. Identify one of the possible locations of the sun on your graph.

*Model Assessment Answer(s):*

\(a\)

\[\begin{array}{c}
\text{Figure P15-2} \\
\end{array}\]

\[\begin{array}{c}
\text{Foci} \\
\end{array}\]

\(b\) The sun could be located at one of two foci: \((-\sqrt{7}, 0)\) or \((\sqrt{7}, 0)\).
COMPETENCY 16

Graph trigonometric functions of the form \( y = A \cdot f(Bx + C) + k \).
- Determine the amplitude of a sine or cosine function from its equation.
- Determine the period of a sine, cosine, or tangent function from its equation.
- Determine the vertical shift of a sine, cosine, or tangent function from its equation.
- Determine the phase shift of a sine, cosine, or tangent function from its equation.
- Sketch by hand the graph of a sine, cosine, or tangent function.

Prior Knowledge:
- Graph linear, polynomial, and radical equations.
- Describe graphs of transformed functions in terms of the transformations applied to parent functions.

Model Assessments

Assessment Example 1:
Given the equation \( y = -3\sin(2x + \pi) - 4 \):

a) Determine the amplitude, period, vertical shift, and phase shift without using a calculator; then sketch the graph by hand in the coordinate plane.

b) Now write the equation of the same graph in terms of cosine instead of sine.
Model Assessment Answer(s):

a) Reflection on x-axis: amplitude = 3, period = π, vertical shift = -4, phase shift = $-\frac{\pi}{2}$.

![Graph of a sine function with amplitude 3, period π, vertical shift -4, and phase shift $-\frac{\pi}{2}$]

b) $y = -3\cos\left(2x + \frac{\pi}{2}\right) - 4$.

Assessment Example 2:

The US Air Force recently launched a satellite to keep tabs on other space craft and also to keep an eye on the growing problem of orbiting space junk. This satellite is orbiting the Earth so that its displacement $D$ north of the equator is given by the equation $D = A\sin(\omega t + \alpha)$.

Sketch two cycles of $D$ as a function of $t$ if $A = 500$ km., $\omega = 3.60$ rad./hr. and $\alpha = 0$.

Model Assessment Answer(s):

$D = 500\sin(3.6t)$.

![Graph of a sine function representing the displacement $D$ over time $t$]
Assessment Example 3:

Graph the following equations, a and b, on separate axes.

a) \[ y = \frac{1}{2} \sin \left( 2x - \frac{\pi}{4} \right). \]

b) \[ y = 3 + \tan \left( 2x - \frac{\pi}{4} \right). \]

Model Assessment Answer(s):

a) [Graph of equation a]

b) [Graph of equation b]
COMPETENCY 17

Given the graph of a trigonometric function, write the equation (sine, cosine, tangent).

Prior Knowledge:
- Identify graphs of periodic functions as graphs of sine, cosine, or tangent functions.
- Identify the period, amplitude, and frequency of sine, cosine, and tangent functions.
- Identify transformations applied to parent functions.

Model Assessments

Assessment Example 1:

The table shows the maximum daily high temperatures for Daytona Beach for month $t$, with $t = 1$ corresponding to January.

<table>
<thead>
<tr>
<th>$t$</th>
<th>Daytona Beach high temperatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64.8</td>
</tr>
<tr>
<td>2</td>
<td>68.4</td>
</tr>
<tr>
<td>3</td>
<td>75.0</td>
</tr>
<tr>
<td>4</td>
<td>81.0</td>
</tr>
<tr>
<td>5</td>
<td>87.5</td>
</tr>
<tr>
<td>6</td>
<td>91.9</td>
</tr>
<tr>
<td>7</td>
<td>93.0</td>
</tr>
<tr>
<td>8</td>
<td>92.5</td>
</tr>
<tr>
<td>9</td>
<td>89.5</td>
</tr>
<tr>
<td>10</td>
<td>82.2</td>
</tr>
<tr>
<td>11</td>
<td>73.9</td>
</tr>
<tr>
<td>12</td>
<td>66.8</td>
</tr>
</tbody>
</table>

a) Plot the points on a graph.

b) Using the data and the graph, find an equation for the high temperature at month $t$.

c) Most people plan their vacations when the high temperatures are between 76° and 82°. The tourism industry needs to begin marketing one month before this period. During what months should they implement their marketing campaign?
Model Assessment Answer(s):

a) Using a graphing calculator, the data is entered into lists. Since the data is for 12 months, the data is repeated in the lists creating a period of 24 months for analysis. Using the SinReg L1, L2, 12, Y1, the resulting equation is:

\[ T(t) = 14.193 \sin(0.520t - 2.065) + 80.447 \text{ for } 1 \leq t \leq 12 \text{ months.} \]

b) The tourism industry needs to run marketing campaigns beginning in February and September.
Assessment Example 2:
Given the graph below, write the equation for the graph as a sine function and as a cosine function.

Model Assessment Answer(s):
\[ y = -3\sin(2x + \pi) - 4, \]

or \[ y = -3\cos\left(2x + \frac{\pi}{2}\right) - 4. \]
COMPETENCY 18

Graph inverse trigonometric functions. (This is a prerequisite skill that is used in Calculus.)

Prior Knowledge:
- Graph sine, cosine, and tangent functions.
- Identify amplitude, period, and phase shift of common trigonometric functions from their equation and their graphs.
- Determine the inverse of functions.
- Identify the domain and range of functions.

Model Assessments

Assessment Example 1:
Graph the following functions:

a) $f(x) = \arcsin 2x$.

b) $f(x) = \tan^{-1} x$.

c) $f(x) = \cos^{-1} \left( \frac{x}{2} \right)$.

Model Assessment Answer(s):

a) $f(x) = \arcsin 2x$. 
b) \( f(x) = \tan^{-1}x. \)

\[
\begin{align*}
\pi/2 & \\
\pi/4 & \\
-\pi/4 & \\
-\pi/2 & \\
-10 & -8 & -6 & -4 & -2 & 2 & 4 & 6 & 8 & 10
\end{align*}
\]

\[
\begin{align*}
\pi & \\
3\pi/4 & \\
\pi/4 & \\
-\pi & \\
-3\pi/4 & \\
-\pi/4 & \\
-\pi/2 & \\
-3\pi/4 & \\
-\pi & \\
-2.5 & -2 & -1.5 & -1 & -0.5 & 0.5 & 1 & 1.5 & 2 & 2.5
\end{align*}
\]

c) \( f(x) = \arccos\left(\frac{x}{2}\right) = \cos^{-1}\left(\frac{x}{2}\right). \)
COMPETENCY 19

Prove trigonometric identities. (This is a prerequisite skill that is used in Calculus.)

Prior Knowledge:
• Define basic trigonometric functions.
• Solve trigonometric equations.
• Apply polynomial factoring techniques.
• Apply the Pythagorean Theorem.

Model Assessments

Assessment Example 1:
Show that: \( \sin(x + y) + \sin(x - y) = 2 \sin x \cos y \).

Model Assessment Answer(s):
\[
\sin(x + y) + \sin(x - y) = 2 \sin x \cos y,
\]
\[
\sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y = 2 \sin x \cos y.
\]

Assessment Example 2:
Show that: \( \frac{\cos x}{\sec x + \tan x} = 1 - \sin x \).

Model Assessment Answer(s):
\[
\frac{\cos x}{\sec x + \tan x} = 1 - \sin x,
\]
\[
\frac{\cos x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = 1 - \sin x,
\]
\[
\frac{\cos x}{\frac{1 + \sin x}{\cos x}} = 1 - \sin x,
\]
\[
\frac{\cos x}{\frac{1 + \sin x}{\cos x}} = 1 - \sin x,
\]
\[
\frac{\cos^2 x}{1 + \sin x} = 1 - \sin x,
\]
\[
\frac{1 - \sin^2 x}{1 + \sin x} = 1 - \sin x,
\]
\[
\frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x} = 1 - \sin x,
\]
\[
1 - \sin x = 1 - \sin x.
\]
Assessment Example 3:

Show that: \[
\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}.
\]

Model Assessment Answer(s):

\[
\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x},
\]

\[
\frac{\sin x(1 - \cos x)}{(1 + \cos x)(1 - \cos x)} = \frac{1 - \cos x}{\sin x},
\]

\[
\frac{\sin x(1 - \cos x)}{1 - \cos^2 x} = \frac{1 - \cos x}{\sin x},
\]

\[
\frac{\sin x(1 - \cos x)}{\sin^2 x} = \frac{1 - \cos x}{\sin x},
\]

\[
\frac{1 - \cos x}{\sin x} = \frac{1 - \cos x}{\sin x}.
\]
COMPETENCY 20

Solve trigonometry equations.

Prior Knowledge:
- Identify the values of trigonometric functions at standard points around the unit circle.
- Determine the values of inverse trigonometric functions given values corresponding to standard points on the unit circle.
- Identify the domain and range of functions.
- Identify the domain and range of inverse trigonometric functions.
- Identify the period of trigonometric functions.
- Convert between angle measures in radians and degrees.

Model Assessments

Assessment Example 1:

a) Solve the equation \( \frac{\cos x}{2} = \frac{\sqrt{3}}{2} \) for \( x \) on the interval \( 0 \leq x < 2\pi \).

b) Find all solutions for the equation \( \sin 2x = \frac{\sqrt{3}}{2} \).

c) Solve the equation \( \sin x + \sin^3 x = 2 \sin x \cos^2 x \) for \( x \) on the interval \( 0 \leq x < 2\pi \).

Model Assessment Answer(s):

a) \( x = \frac{\pi}{3} \)

b) There are two families of solutions: \( x = \frac{\pi}{6} + k\pi \) and \( x = \frac{\pi}{3} + k\pi \), where \( k \) is an integer.

c) \( x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \).

Real World Application Reference:
- Marketing, Sales, and Service
**Assessment Example 2:**
Solve the following equations (rounded to two decimal places, answers in degrees):

a) \( \sin x = 0.3 \) for \( 0^\circ \leq x < 360^\circ \).

b) \( \tan x = 1.5 \) for \( 0^\circ \leq x < 360^\circ \).

**Model Assessment Answer(s):**

a) \( x = \sin^{-1} 0.3 \approx 17.46^\circ \) or \( 162.54^\circ \).

b) \( x = \tan^{-1} 1.5 \approx 56.31^\circ \) or \( 236.31^\circ \).

**Assessment Example 3:**
A snowboard manufacturer finds that sales of their newest model of snowboard follows a seasonal pattern, with sales given by:

\[
S = 6320 + 2850 \cos\left(\frac{\pi t}{6}\right),
\]

where \( S \) is the number of boards sold during a given month \( t \), and \( t = 1 \) corresponds to January. As local stores develop their inventory, they need to estimate sales by month. For which months of the year are sales greater than 5,000 boards?

**Model Assessment Answer(s):**
Solve the inequality \( 6320 + 2850 \cos\left(\frac{\pi t}{6}\right) > 5000 \).

\[
2850 \cos\left(\frac{\pi t}{6}\right) > -1320,
\]

\[
\cos\left(\frac{\pi t}{6}\right) > -0.46316.
\]

Find critical values (i.e., when \( \cos\left(\frac{\pi t}{6}\right) = -0.46316 \)).

These will be when \( \frac{\pi t}{6} = \cos^{-1}(-0.46316) \),

or when \( t = \frac{6}{\pi} \cos^{-1}(-0.46316) \).

This gives us two values of \( t \), \( t = 3.9 \) and \( t = 8.1 \). When \( t \) is between these two values (for example, when \( t = 6 \)) we get a false statement, so we must want the values of \( t \) that fall outside of the interval \((3.9, 8.1)\). Therefore, we know that sales are greater than 5,000 boards when \( t < 3.9 \) and when \( t > 8.1 \).

That would mean that in the months of January, February, March, September, October, November, and December, sales will be greater than 5,000 boards.
COMPETENCY 21

Solve for all unknown parts of a given triangle:

a) by using right triangle trigonometry
b) by using the Law of Sines
c) by using the Law of Cosines

Prior Knowledge:
- Define sine, cosine, and tangent in terms of the sides of a right triangle.
- Solve polynomial equations.
- Use the Pythagorean Theorem to find the missing lengths of a side of a right triangle.
- Determine the reference angle given its sine or cosine value.

Model Assessments

Assessment Example 1:
Find the measure of \( \angle A \), the length of AC, and the length of BC.

Model Assessment Answer(s):
Using Triangle Sum: \( \angle A = 60^\circ \). Using Special Triangles: \( a = 5\sqrt{3}; \ b = 10 \).

Using trigonometry: \( \tan 30^\circ = \frac{5}{a}; \ a \approx 8.6603 \) and \( \sin 30^\circ = \frac{5}{b}; \ b = 10 \).
Assessment Example 2:
Find the measures of all of the angles in the triangle below.

Model Assessment Answer(s):
Using Law of Cosines: \( A \approx 31.00^\circ \). Using Law of Cosines again: \( B \approx 121.97^\circ \).
Using Triangle Sum: \( C \approx 27.03^\circ \).

Assessment Example 3:
Find the measure of \( \angle C \), \( \angle B \), and the length of \( BC \).

Model Assessment Answer(s):
Using Law of Cosines: \( a \approx 20.12 \). Using Law of Cosines again: \( B \approx 93.69^\circ \).
Using Triangle Sum: \( C \approx 66.19^\circ \).
Assessment Example 4:

A search and rescue team needs to estimate the height of a cliff in order to plan their rescue attempt. To estimate the height, two sightings were taken 50 feet apart. The angles of elevation were 40 degrees and 31 degrees. Find the height of the cliff, $h$ (see figure).

![Figure PC21-4](image)

**Model Assessment Answer(s):**

Using Triangle Sum: The angles of the obtuse triangle are $9^\circ$ and $140^\circ$.

Using Law of Sines: $a \approx 205.45$ feet.

Using Right Triangle Trig.: $h \approx 105.81$ feet.
COMPETENCY 22

Convert between polar and rectangular coordinates.

Prior Knowledge:
- Convert between angle measures in degrees and radians.
- Use the Pythagorean Theorem to find a missing side of a right triangle.
- Use trigonometry to determine unknown sides or angles in a right triangle.
- Compute the values of trigonometric functions and inverse trigonometric functions at various standard points.

Model Assessments

Assessment Example 1:
Convert the following Cartesian (rectangular) points to polar coordinates \((r, \theta)\):

a) \((-3, 3)\).
b) \((2, -2\sqrt{3})\).

Convert the following polar coordinates to Cartesian (rectangular) coordinates:

c) \(\left(8, \frac{5\pi}{3}\right)\).
d) \((-4, 30^\circ)\).
Given an overlap of the polar grid and the rectangular grid, four equations arise to do these conversions:

\[ x^2 + y^2 = r^2 \quad \text{and} \quad x = r \cos \theta \]
\[ \tan \theta = \frac{y}{x} \quad \text{and} \quad y = r \sin \theta \]

Rectangular to polar coordinates:

a) \((-3, 3)\) would be in QII. Two possible polar answers for \((-3, 3)\) are
\[
(r, \theta) = \left(-3\sqrt{2}, -\frac{\pi}{4}\right) \text{ or } \left(3\sqrt{2}, \frac{3\pi}{4}\right).
\]

b) \((2, -2\sqrt{3})\) would be in QIV. Two possible polar answers for \((2, -2\sqrt{3})\) are
\[
(r, \theta) = (4, -60^\circ) \text{ or } (-4, 120^\circ).
\]

Polar to rectangular coordinates:

\(8, \frac{5\pi}{3}\):
\[
(x, y) = (4, -4\sqrt{3})
\]

d) \((-4, 30^\circ)\):
\[
(x, y) = (-2\sqrt{3}, -2)
\]
Assessment Example 2:

A forest survey technician is completing a survey of diseased trees. He plans to plot the location of all infected trees on a rectangular grid. Standing at the origin, he takes a bearing (40° E of N) on the nearest infected tree. He measures the distance from his location to the tree to be 45 meters. At what location should he mark the tree in Cartesian coordinates on his grid?

Model Assessment Answer(s):

Using the angle of 50° made with the x-axis: \((x, y) = (28.9 \text{ m}, 34.5 \text{ m}).\)
COMPETENCY 23

Graph equations written in polar form.

Prior Knowledge:

- Convert between angle measures in degrees and radians.
- Compute the values of trigonometric functions and inverse trigonometric functions at various standard points.
- Graph linear, polynomial, radical, and trigonometric functions.

Model Assessments

Assessment Example 1:
Graph the following equations:

a) \( r = -2\cos\theta \).

b) \( r = 2 - 4\cos\theta \).

c) \( r = 4 + 4\sin\theta \).

Model Assessment Answer(s):

\( r = \cos\theta \) has symmetry over the polar axis, and \( r = \sin\theta \) has symmetry over the \( \frac{\pi}{2} \) axis.

For a and b, use this table of values:

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0</th>
<th>( \frac{\pi}{4} )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \frac{3\pi}{4} )</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos\theta )</td>
<td>1</td>
<td>0.707</td>
<td>0</td>
<td>-0.707</td>
<td>-1</td>
</tr>
<tr>
<td>( r = -2\cos\theta )</td>
<td>-2</td>
<td>-1.414</td>
<td>0</td>
<td>1.414</td>
<td>2</td>
</tr>
<tr>
<td>( r = 2 - 4\cos\theta )</td>
<td>-2</td>
<td>-0.828</td>
<td>2</td>
<td>4.828</td>
<td>6</td>
</tr>
</tbody>
</table>
a) 

![Graph of a circle with radius 1 centered at the origin.](image)

b) 

![Graph of a circle with radius 7 centered at the origin.](image)
For c, use this table of values:

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$-\pi/2$</th>
<th>$-\pi/4$</th>
<th>0</th>
<th>$\pi/4$</th>
<th>$\pi/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td>-1</td>
<td>-0.707</td>
<td>0</td>
<td>0.707</td>
<td>1</td>
</tr>
<tr>
<td>$r = 4 + 4 \sin \theta$</td>
<td>0</td>
<td>1.2</td>
<td>4</td>
<td>6.8</td>
<td>8</td>
</tr>
</tbody>
</table>

**Assessment Example 2:**

A fire investigator is analyzing a recent brush fire in which a perpetrator propelled a fiery object into a field. The path of the object is given by the equation:

$$r(\theta) = \frac{20}{1+\sin \theta},$$

where $r$ is measured in meters. To help the investigator locate the launch point, graph the path of the object for $0 \leq \theta \leq \pi$. 
Model Assessment Answer(s):

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0</th>
<th>$\pi/4$</th>
<th>$\pi/2$</th>
<th>$3\pi/4$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td>0</td>
<td>0.707</td>
<td>1</td>
<td>0.707</td>
<td>0</td>
</tr>
<tr>
<td>$r = \frac{20}{1+\sin \theta}$</td>
<td>20</td>
<td>11.7</td>
<td>10</td>
<td>11.7</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure P23-2
COMPETENCY 24

Graph equations written in parametric form.

Prior Knowledge:
- Graph linear, polynomial, radical, and trigonometric functions.
- Graph polar equations.
- Solve problems involving composition of functions.
- Identify geometric properties of conic sections from their quadratic equations.
- Graph conic sections including translations.
- Simplify equations using trigonometric identities.
- Evaluate trigonometric functions.

Model Assessments

Assessment Example 1:
Graph; show orientation:

a) \( x = \cos t, \ y = \sin t \).

b) \( x = 2t - 3 \) for \( 0 \leq t < 10 \) and \( y = -t + 5 \).

Model Assessment Answer(s):

a) Points are plotted and arrows indicate the direction in which the points are being swept out.
Assessment Example 2:

Graph equations in parametric form.

A combine harvests corn from a field and the kernels are removed from the cob and ejected out of the back in a stream. The stream exits the combine at a height of 10 feet above the ground with an initial speed of 25 feet per second at an angle of elevation of 20 degrees relative to the ground. The stream of corn has to land in a 14-foot wagon towed behind the combine. The top of the wagon is 8 feet above the ground and the bed of the wagon is 2 feet above the ground.

a) Write a set of parametric equations that model the path of the stream.

b) Graph the path of the stream.

c) How far behind the combine must the wagon follow in order to catch the corn in the middle of the trailer?
**Model Assessment Answer(s):**

The drawing illustrates what is occurring (not necessarily drawn to scale):

![Diagram](image)

a) In the x-direction, ignoring friction, \( D = rt \). So \( x \) is equal to the speed in the x-direction multiplied by time; \( x = 25\cos20^\circ t \), measured in feet.

In the y-direction, we must take into consideration the force of gravity pulling on the corn as well as the speed acting in the y direction and the elevation from which the corn is being propelled. So \( y = -\frac{1}{2}gt^2 + 25(\sin20^\circ)t + 10 \) measured in feet, where \( g \) is the acceleration due to gravity (32.2 ft./sec.²).

b) ![Chart](image)

c) Solving the equation in the y-direction for the height of the bed of the wagon, the corn would hit the bed of the trailer about 23.9 feet behind the combine. Therefore, the trailer should be about 16.5 feet behind the wagon for the corn to clear the sides of the wagon and land near the middle.
COMPETENCY 25

Given the components of a two-dimensional vector, determine the magnitude and direction of the vector.

Prior Knowledge:
• Solve problems involving exponents and square roots.
• Apply the Pythagorean Theorem.
• Compute inverse trigonometric functions.

Model Assessments

Assessment Example 1:
Find the magnitude and direction of the following vectors:
\[-7, 7\sqrt{3}\] or \[-7\mathbf{i} + 7\sqrt{3}\mathbf{j}\] or \[-7\mathbf{i} + 7\sqrt{3}\mathbf{j}\], where \(\mathbf{i}, \mathbf{j}\) or \(\hat{\mathbf{i}}, \hat{\mathbf{j}}\) are the unit vectors in the x and y directions, respectively.

Model Assessment Answer(s):
Magnitude: 14.
Direction: \(\theta = -60^\circ\) in standard position. However, since the vector is going in the opposite direction, the direction angle in standard position is \(\theta = 120^\circ\).

Assessment Example 2:
Let point \(P\) be \(P = (-2, 5)\) and let point \(Q\) be \(Q = (3, 2)\).
Express \(\mathbf{PQ}\) in component form.

Model Assessment Answer(s):
\[\mathbf{PQ} = 5\mathbf{i} - 3\mathbf{j} = (5, -3)\.]
**Assessment Example 3:**

What is the magnitude and direction of the vector \( \mathbf{PQ} \) above?

**Model Assessment Answer(s):**

Magnitude: \( |PQ| = \sqrt{34} \).

Direction: \( \theta = \tan^{-1} \left( \frac{-3}{5} \right) \approx -30.96^\circ \) in standard position.

**Assessment Example 4:**

A crop duster (small airplane) is taking off. There is a 179-foot-tall grain storage silo that is 355 feet away from the take-off point. Find the angle of elevation the pilot needs in order to clear the silo by 10 feet, and write an expression for the departure vector. How far has the crop duster traveled when it is directly above the silo?

**Model Assessment Answer(s):**

Crop duster path vector: \( \mathbf{v} = 355\mathbf{i} + 189\mathbf{j} \).

Distance traveled: \( |\mathbf{v}| = \sqrt{355^2 + 189^2} \approx 402.18 \text{ ft.} \)

Angle of elevation: \( \theta_r = \tan^{-1} \left( \frac{189}{355} \right) \approx 28.03^\circ \).
**COMPETENCY 26**

*Perform the vector operations of addition, subtraction, and scalar multiplication.*

**Prior Knowledge:**
- Use matrix notation.
- Identify the dimensions of a matrix.
- Identify the positions of each entry in a matrix.

**Model Assessments**

**Assessment Example 1:**
Given: \( \mathbf{a} = \langle -2, 5 \rangle, \mathbf{b} = \langle 3, 7 \rangle, \mathbf{c} = \langle -1, -4 \rangle \), find:

a) \( \mathbf{a} + \mathbf{b} \).

b) \( 2\mathbf{a} - 3\mathbf{c} \).

c) \( 7\mathbf{c} + \mathbf{b} - 5\mathbf{a} \).

**Model Assessment Answer(s):**

a) \( \mathbf{a} + \mathbf{b} = \langle 1, 12 \rangle \).

b) \( 2\mathbf{a} - 3\mathbf{c} = \langle -1, 22 \rangle \).

c) \( 7\mathbf{c} + \mathbf{b} - 5\mathbf{a} = \langle 6, -46 \rangle \).

**Assessment Example 2:**
A plane flies 250 miles per hour on a compass heading or bearing of 300 degrees. The air is moving with a wind speed of 50 miles per hour on a bearing of 195 degrees. Find the plane’s resultant velocity (speed and bearing) by adding these two velocity vectors.

**Real World Application Reference:**
- Transportation
Model Assessment Answer(s):

Bearing is from north, going clockwise. The two vectors are drawn below. \( \mathbf{P} \) will be the vector of the plane with angle measured in standard position; \( \mathbf{w} \) will be the wind vector with angle in standard position.

Resultant vector:

\[
\mathbf{R} = \mathbf{P} + \mathbf{w} = \begin{pmatrix} -229.45 \\ 76.70 \end{pmatrix} \text{ mph. Actual speed: } |\mathbf{P}| = 241.93 \text{ mph.}
\]

Heading in standard position: \( \theta_r \approx 18.5^\circ \).

Bearing: \( 270^\circ + 18.5^\circ = 288.5^\circ \) from north, clockwise.
**COMPETENCY 27**

*Use factorial notation.*

**Prior Knowledge:**
- Use simple mathematical concepts such as multiplication and order of operations.

**Model Assessments**

**Assessment Example 1:**
Five friends went to the movies. In how many different arrangements could they be seated in a row together?

*Model Assessment Answer(s):*

\[ 5! = 120 \text{ different arrangements are possible.} \]

**Assessment Example 2:**
There are 100 songs on your iPod. If the songs are shuffled with no repeats, in how many different orders could they be played?

*Model Assessment Answer(s):*

\[ 100! \text{ different arrangements (that would be approximately } 9.333 \times 10^{157} \text{).} \]

**Assessment Example 3:**
A concert promoter has 20 bands participating in a weekend concert event. How many different orders are possible for the bands during the concert?

*Model Assessment Answer(s):*

\[ 20! = 2.43 \times 10^{18} \text{ different orders possible.} \]

**Real World Application Reference:**
- Arts, Media, and Entertainment
COMPETENCY 28

Expand binomial expressions. (This is a prerequisite skill that is used in Calculus.)

Prior Knowledge:
• Multiply binomials and polynomials.
• Simplify polynomial expressions.
• Apply properties of exponents.
• Compute combinations.

Model Assessments

Assessment Example 1:
Expand \((2a - b)^5\).

Model Assessment Answer(s):
Using the Binomial Theorem, we get:
\[
(2a - b)^5 = 32a^5 - 80a^4b + 80a^3b^2 - 40a^2b^3 + 10ab^4 - b^5.
\]

Assessment Example 2:
In the expansion of \((x + y)^{16}\), what is the term containing \(x^7\)?

Model Assessment Answer(s):
11440\(x^7y^9\).

Assessment Example 3:
Simplify \(\frac{(x+h)^4 - x^4}{h}\) using the Binomial Theorem.

Model Assessment Answer(s):
\[4x^3 + 6x^2h + 4xh^2 + h^3.\]
Statistics (Non-STEM) Competencies
COMPETENCY 1

Determine whether a variable is categorical or quantitative, given a situation.

Prior Knowledge:
- Identify the variable in an experiment.
- Organize, represent, and interpret numerical and categorical data.

Model Assessments

Assessment Example 1:
As a member of the math department at the local community college, you need to make a presentation on the data collected about the students in the statistics classes at the school. In order to perform any kind of analysis on the data, you have to identify what type of data you have; this dictates the methods of analysis that can be employed. Determine whether the following variables are categorical or quantitative if students in a statistics class are the observational units:

a) how many classes a student takes  
b) the students’ eye color  
c) average weekly sleep time

Model Assessment Answer(s):
- a) quantitative  
- b) categorical  
- c) quantitative

Assessment Example 2:
As the head of nursing for the hospital, you have collected data regarding the nurses and their functions. You are analyzing this data, and in order to perform the analyses correctly, you have to know the types of variables that are represented. Determine whether each variable is categorical or quantitative if nurses in a hospital are the observational units:

a) the number of patients under a nurse’s care  
b) the amounts of antibiotics administered to patients  
c) the year a nurse began working at the hospital

Real World Application Reference:
- Education, Child Development, and Family Services  
- Health Science and Medical Technology  
- Public Services
Model Assessment Answer(s):

a) quantitative
b) quantitative
c) categorical

Assessment Example 3:

Your city is seeking areas in the budget where costs can be reduced. One area identified for potential cuts in the budget is the fire department. Data have been collected to analyze this possibility. In order to determine the proper method of analysis, you (as the analyst) have to identify the variable types. Determine whether each variable is categorical or quantitative if firefighters at a fire station are the observational units:

a) the number of fires to which a firefighter has been called
b) the days of the week a firefighter reports to a fire station
c) the street numbers of buildings to which a firefighter has been called

Model Assessment Answer(s):

a) quantitative
b) categorical
c) categorical
COMPETENCY 2

Identify the sampling method used (random, systematic, convenience, stratified, cluster, or voluntary response), given a scenario in which data were collected from a population.

Prior Knowledge:

- Read and interpret polls.
- Interpret surveys and their results.
- Define random samples.

Model Assessments

Assessment Example 1:
Identify which type of sampling is used: random, systematic, convenience, stratified, cluster, or voluntary response:

a) A local college conducts a study on student cheating and plagiarism by randomly selecting 30 different classes and giving all of the students in each of those classes a survey form to complete.

b) A television station wants to solicit its viewers’ opinion on the current state of the economy by asking them to go to its Web site to complete a survey.

c) A marketing executive for a television network is planning to survey 1,500 of its young viewers. The 1,500 viewers will be randomly selected from each age group of 8–12, 13–17, and 18–22.

Model Assessment Answer(s):

a) cluster

b) voluntary response

c) stratified

Real World Application Reference:

- Health Science and Medical Technology
- Marketing, Sales, and Service
**Assessment Example 2:**
Identify which type of sampling is used: random, systematic, convenience, stratified, cluster, or voluntary response:

a) A cell phone provider conducts a customer satisfaction survey and uses a computer to randomly generate 1,000 telephone numbers to call.

b) A military base sets up a sobriety checkpoint in which every 10th driver is stopped and interviewed.

c) A statistics student conducts research on the amount of time people spend texting per day by polling her family, friends, and classmates.

*Model Assessment Answer(s):*

a) random

b) systematic

c) convenience

---

**Assessment Example 3:**
Identify which type of sampling is used: random, systematic, convenience, stratified, cluster, or voluntary response:

a) A university medical center researcher surveys all cancer patients in each of 25 randomly selected hospitals.

b) A radio station conducts a popularity survey and asks its listeners to call in stating whether they are on Team X or Team Y.

*Model Assessment Answer(s):*

a) cluster

b) voluntary response
COMPETENCY 3

Identify the basic terminology of experimental design used in a given scenario.

Prior Knowledge:

• Identify the different methods of sampling data.
• Identify the variable that is measured in an experiment.

Model Assessments

Assessment Example 1:

As video gaming becomes more popular, researchers are interested in the effect of low lighting on eyesight. One such research group conducts an experiment on lab rats in which extended exposure to low lighting will be administered. Fifty white lab rats are available for testing. Researchers number the rats from 01 to 50 and use a random digit table to select 25 rats. These 25 will receive the treatment; the remaining 25 will be the control group. Results will be compared at the end of the experiment. Which experimental design did the researchers use?

Model Assessment Answer(s):

completely randomized design

Assessment Example 2:

A manufacturer of golf clubs field tests a new product. The company would like to advertise that using this product will increase the effectiveness of a golfer’s swing, but must obtain such results first. One hundred volunteers are available to test the new product. A researcher decides to have each volunteer try a course twice, once with the new product and once without. The order in which each golfer receives the treatment is decided randomly. Which experimental design does the researcher use?

Model Assessment Answer(s):

matched-pairs design
**Assessment Example 3:**

A new pesticide is to be tested on a large sample of green beans. From past experience, researchers have reason to believe that a new hybrid type of green bean might react differently to pesticides than the traditional green bean. Therefore, all plants of the hybrid type of green bean are grown together in one plot of land; the regular plants are grown together in a separate plot. The pesticide is applied to half of each plot; the other half is left pesticide-free (for control purposes). After a few weeks, results are compared. Which experimental design did the researchers use?

**Model Assessment Answer(s):**

block design
COMPETENCY 4

Identify biases in experimental designs or in the creation of a sample.

Prior Knowledge:
- Identify the different methods of sampling.
- Recognize the type of experimental design employed in a given situation.
- Identify the variable that is measured in the experiment.
- Identify data that represent sampling errors.
- Explain why a sample might be biased.

Model Assessments

Assessment Example 1:
In order to determine whether regular exercise reduces the risk of a heart attack, two methods are employed. Which design will produce more accurate data and why?

Design 1: A researcher finds 2,000 middle-aged men who exercise regularly and have not had heart attacks. She matches each with a similar man who does not exercise regularly, and she follows both groups for 5 years. After 5 years, she compares the number of heart attacks in both groups.

Design 2: Another researcher finds 4,000 middle-aged men who have not had heart attacks and are willing to participate in a study. She randomly assigns 2,000 of the men to a regular program of supervised exercise. The other 2,000 continue their usual habits. The researcher follows both groups for 5 years.

Model Assessment Answer(s):
Design 2 will produce more accurate data because it is a designed experiment and can establish causal relationships, whereas Design 1 is an observational study and can be affected by other factors.
Assessment Example 2:

A questionnaire was sent to 10,000 people asking about ice cream. Five thousand people responded to the questionnaire. Three thousand of the respondents indicated that they “love chocolate ice cream.” It is concluded that 60% of people love chocolate ice cream. In this situation, what is wrong with the survey method and conclusion?

Model Assessment Answer(s):

This is a voluntary response sample. The survey is based on voluntary, self-selected responses and, therefore, has potential for bias. In addition, the 5,000 people who did not respond may not like chocolate ice cream.

Assessment Example 3:

A researcher wished to gauge public opinion on gun control. He randomly selected 1,000 people from among registered voters and asked them the following question: “Do you believe that gun control laws that restrict the ability of Americans to protect their families should be eliminated?” Identify how the researcher’s methods could be improved.

Model Assessment Answer(s):

There is response bias because the question leads respondents to answer in a certain way. A more neutral way to phrase the question would be: “Do you believe that gun control laws should be strengthened, weakened, or left in their current form?”
COMPETENCY 5

Determine an appropriate experimental design for a given scenario.

Prior Knowledge:

• Determine whether a variable is categorical or quantitative.
• Identify different methods of sampling.
• Identify the basic terminology of experimental design.

Model Assessments

Assessment Example 1:

As video gaming becomes more popular, researchers are interested in the effect of low lighting on eyesight. One such research group conducts an experiment on lab rats in which extended exposure to low lighting will be administered. Fifty white lab rats are available for testing. Determine the appropriate experimental design for this scenario.

Model Assessment Answer(s):

Researchers should use a completely randomized design. Number the rats from 01 to 50 and use a random digit table to select 25 rats. These 25 will receive the treatment; the remaining 25 will be the control group. Results should be compared at the end of the experiment.

Assessment Example 2:

A manufacturer of golf clubs is field-testing a new product. The company would like to advertise that using this product will increase the effectiveness of a golfer's swing, but must obtain such results first. One hundred volunteers are available to test the new product. Determine the appropriate experimental design for this scenario.

Model Assessment Answer(s):

Researchers should use a matched-pairs design at a driving range. Have each volunteer hit 10 golf balls with his/her own equipment and 10 golf balls with the new product, measuring the ball’s distance for each hit. The order in which each golfer receives the treatment should be decided randomly. This pairing allows researchers to collect data on the product’s effectiveness.
Assessment Example 3:
A new pesticide is to be tested on a large sample of green beans. From past experience, researchers have reason to believe that a new hybrid type of green bean might react differently to pesticides than the traditional green bean. Determine the appropriate experimental design.

Model Assessment Answer(s):
Since there has been a characteristic identified ahead of time that is expected to affect results, researchers should use a block design. Therefore, all the plants of the hybrid type of green bean should be grown together in one plot of land and the regular plants grown together in a separate plot. The pesticide should be applied to half of each plot; the other half should be left pesticide-free (for control purposes). After a few weeks, results should be compared.
COMPETENCY 6

Construct and describe an appropriate visual display given a data set (histogram, bar graph, stem plot, scatter plot, box and whisker plot, and pie chart).

Prior Knowledge:
- Calculate mean and median.
- Calculate percentage.
- Graph points on a coordinate plane.

Model Assessments

Assessment Example 1:
The number of years of teaching experience of teachers at a local high school is listed below. Construct an appropriate visual display of the data:

0, 3, 8, 14, 26, 0, 4, 9, 17, 23, 4, 9, 1, 18, 21, 2, 21, 7, 32, 9.

Model Assessment Answer(s):
One of the following would be appropriate:

a) [Box plot image]
b) histogram

<table>
<thead>
<tr>
<th>Number of years of experience</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–5</td>
<td>7</td>
</tr>
<tr>
<td>6–11</td>
<td>5</td>
</tr>
<tr>
<td>12–17</td>
<td>2</td>
</tr>
<tr>
<td>18–23</td>
<td>4</td>
</tr>
<tr>
<td>24–29</td>
<td>1</td>
</tr>
<tr>
<td>30–35</td>
<td>1</td>
</tr>
</tbody>
</table>

Teaching experience at local high school

![Histogram showing teaching experience at local high school](image)
**Assessment Example 2:**

Suppose 200 seniors at the local high school have after-school jobs. The type of jobs and respective number of students who hold them are detailed below. Construct an appropriate visual display of the data.

- Retail, 65
- Food, 72
- Manual labor, 8
- Health and human services, 38
- Business/office, 5
- Other, 12

**Model Assessment Answer(s):**

One of the following would be appropriate:

a) bar graph

![Bar graph of jobs held by seniors at the local high school](image)
b) pie chart

Jobs held by seniors at the local high school

Assessment Example 3:

Data has been gathered at a book store for many years. Analysts want to study how an author’s popularity has changed over the years. Below is a table that provides data of interest. Create a graph to show the author’s popularity over time.

<table>
<thead>
<tr>
<th>Number of years since author’s debut book</th>
<th>Number of fans at author’s book signing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>398</td>
</tr>
<tr>
<td>1</td>
<td>284</td>
</tr>
<tr>
<td>2</td>
<td>189</td>
</tr>
<tr>
<td>3</td>
<td>152</td>
</tr>
<tr>
<td>4</td>
<td>102</td>
</tr>
<tr>
<td>5</td>
<td>77</td>
</tr>
<tr>
<td>6</td>
<td>52</td>
</tr>
<tr>
<td>7</td>
<td>39</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>109</td>
</tr>
<tr>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>
Model Assessment Answer(s):

Author's popularity over time

Number of years since the author's debut book

Number of fans at an author's book signing
COMPETENCY 7

Identify which measure of the center (mean, median, or mode) is most appropriate for a given situation and what the relationship between the mean, median, and mode indicates about the distribution of the data.

Prior Knowledge:

- Calculate the mean, median, and mode of a data set.
- Identify the shape of different data distributions from their graph (i.e., histogram, curve, scatter plot, etc.).

Model Assessments

Assessment Example 1:

The coach of a little league team is suspicious of the ages of his rival team based on the players’ heights. The coach obtains the roster of the opposing team and finds these reported heights (rounded to the nearest inch):

50, 69, 50, 55, 50, 54, 49, 47, 50, 49, 70, 56, 58, 50, 57, 50.

a) Which measure of center supports the coach’s suspicion? Explain and support your reasoning with numerical proof.

b) Which measure(s) of center defend(s) the opposing team’s legitimacy of their players’ ages based on their heights? Explain your reasoning with numerical evidence.

Model Assessment Answer(s):

a) mean = 54.0 inches, median = 50.0 inches, mode = 50 inches

The mean supports the coach’s suspicion. The mean is a nonresistant measure of center and is being pulled a full 4 inches higher than both the median and mode. This results from two unusually tall kids; the distribution of heights is skewed right, or positively skewed. Four inches in height difference could make an impact on players’ speed, strength, and athletic ability.
b) mean = 54.0 inches, median = 50.0 inches, mode = 50 inches

Both the median and the mode help defend the opposing team's players.

The mean height is fairly high, but the median demonstrates a more typical height. The fact that the median is 50.0 inches — a full 4 inches shorter than the mean — shows that the typical player on the team is about that height. In addition, the fact that the mode also is 50 inches emphasizes the height of the team as not being unusually tall. More players on the team are 50 inches tall than any other height. Since the most frequently occurring (6 players) height is 50 inches, there would be little difference in speed, strength, or athletic ability in these children based on height.

Assessment Example 2:

A district attorney is running for re-election in your city. His campaign strategy is to focus on his “tough on crime” approach by providing data about the amount of time the convicts he prosecuted have spent in jail. Jail times from last year’s court cases were (in months):

4, 18, 7, 2, 0.5, 12, 3, 6, 6, 9, 6, 10, 84, 6, 1, 2, 8, 6, 8.

Which measure of center should the district attorney choose to help his “tough on crime” appearance the most? Support your answer with numerical evidence.

Model Assessment Answer(s):

mean = 10.23 months, median = 6.00 months, mode = 6 months

The mean would give the appearance that criminals prosecuted by this attorney receive longer jail sentences. The mean is a nonresistant measure of center, and is, therefore, pulled higher than the median (the more “typical” jail time) by the 18- and 84-month sentences. To demonstrate just how easily influenced the mean is, one might look at the average jail sentence with the 84-month value removed from the data. In so doing, the mean is now approximately 6.34 months, which is quite a difference! Comparing the median and mode with the mean value also indicates that the distribution of the data is skewed positively (or skewed right), which also shows the effect of those two sentences on the mean value.
Assessment Example 3:

A company that manufactures energy-saving light bulbs for lamps guarantees 3,000 hours of use for each bulb. Every major run of bulbs from the previous 6-month period was sample tested, and the bulb failures are listed in the table below:

<table>
<thead>
<tr>
<th>Date</th>
<th>Batch number</th>
<th>Number of bulbs tested</th>
<th>Number of bulb failures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/15</td>
<td>2381</td>
<td>20,000</td>
<td>50</td>
</tr>
<tr>
<td>1/30</td>
<td>2549</td>
<td>20,000</td>
<td>95</td>
</tr>
<tr>
<td>2/15</td>
<td>2711</td>
<td>20,000</td>
<td>55</td>
</tr>
<tr>
<td>2/28</td>
<td>2908</td>
<td>20,000</td>
<td>61</td>
</tr>
<tr>
<td>3/15</td>
<td>3175</td>
<td>20,000</td>
<td>57</td>
</tr>
<tr>
<td>3/30</td>
<td>3318</td>
<td>20,000</td>
<td>55</td>
</tr>
<tr>
<td>4/15</td>
<td>3560</td>
<td>20,000</td>
<td>63</td>
</tr>
<tr>
<td>4/30</td>
<td>3795</td>
<td>20,000</td>
<td>79</td>
</tr>
<tr>
<td>5/15</td>
<td>3953</td>
<td>20,000</td>
<td>55</td>
</tr>
<tr>
<td>5/30</td>
<td>4150</td>
<td>20,000</td>
<td>72</td>
</tr>
<tr>
<td>6/15</td>
<td>4308</td>
<td>20,000</td>
<td>54</td>
</tr>
<tr>
<td>6/30</td>
<td>4511</td>
<td>20,000</td>
<td>57</td>
</tr>
</tbody>
</table>

Consider the number of bulb failures. Calculate the mean, median, and mode of the number of failures per run. Explain why the mean is significantly higher than the other measures. As a manager of manufacturing, does the data support a major overhaul of the process in order to reduce failures?

*Model Assessment Answer(s):*

The mean is 753/12 = 62.8 failures.
The median is 57.0.
The mode is 55 (the most frequently occurring number).

The mean is significantly higher because of one outstanding run of failures. On 1/30, there were almost twice as many failures as the previous run. There were also two other runs with high numbers of failures, which occurred on 4/30 and 5/30. It seems that the problems occur in the second half of the month (in 5 out of 6 months). A major overhaul does not seem warranted, yet there should be some investigation into any process that seems to generate more failures at the end of the month with frequency. Looking at the distribution of the data, which is skewed positively (or skewed right), also indicates that there is something affecting the data from the second half of the month.
Assessment Example 4:

The administration of a local high school district is considering a push to increase enrollment in AP (Advanced Placement) courses at all school sites. The administration is basing their initiative on the current average AP test passing rate of their students. The administrators hire you as a statistical advisor to help them make the right decision. What would you advise them to consider in terms of measures of center before making their decision?

Model Assessment Answer(s):

Advise the administrator to look at the median AP test score as well as the mode. These measures of center are both resistant to outliers and, therefore, can provide a more “honest” look at AP test scores around the district. For example, if the median score was a 1 (on a 1-5 scale) and some students had scored a 5, the mean would be pulled higher than 1. This would give a false sense of achievement. The effect of the few 5s on the mean value may not be that great, but it is something to consider. By looking at the mode, the administrator would find the score received by the majority of the students taking the test. The mode can be revealing and might show that there should be more of a push to improve the scores for the students currently enrolled before pushing to add more students. At the very least, showing the administrator the different measures of center can help him/her to make an educated decision. A comparison of the mean, median, and mode also would reveal the shape of the distribution of the data, indicating whether or not a small group of high scores may be adversely affecting the average, giving the administrators a false picture of the true results.
COMPETENCY 8

Determine standard deviation from multiple representations of data (table, graph, and word problems) and explain standard deviation in the context of a given situation.

Prior Knowledge:
• Calculate the mean of a data set.
• Recognize and apply summation notation.

Model Assessments

Assessment Example 1:
A student is concerned that her car is not running optimally and decides to collect some data. She measures the miles per gallon she gets each month for 12 months. Below is the gathered data.

29, 28, 29, 37, 27, 30, 30, 31, 32, 30, 28.

After doing some research, she finds that an optimally running car has a standard deviation of 1.6 miles per gallon. Is her car running optimally based on the standard deviation? What would the standard deviation be if she removed the outlier in her data set? Explain.

Model Assessment Answer(s):
According to the data, the sample standard deviation is 2.6 mpg. (which indicates her car is not running optimally). However, she sees that by removing the 37 mpg. data point, \( s = 1.5 \), which could change her conclusion. Therefore, she decides to look more closely at the 37 mpg. outlier.

Assessment Example 2:
North High School is interested in the spread of North fans from game to game for junior varsity (JV) and varsity (V) football. The team manager counts the number of fans for 10 at-home games. The counts are given as follows.

JV fans: 95, 75, 42, 61, 79, 87, 95, 108, 109, 129;

Varsity fans: 215, 201, 186, 175, 192, 184, 199, 213, 220, 232.

Determine if standard deviation is an appropriate choice of measurement of spread and compare the spread of the two distributions of fans. Which team appears to have a larger standard deviation of fans attending the games?

Model Assessment Answer(s):
A histogram or box plot for each data set reveals the data is roughly normal. Once normality is verified, the calculated sample standard deviations are \( s(JV) = 25.2 \) and \( s(V) = 18.1 \). The varsity team has a tighter spread of fans attending the game.

Real World Application Reference:
• Hospitality, Tourism, and Recreation
• Manufacturing and Product Development
**Assessment Example 3:**

Company A and Company B manufacture tractor brakes. Both companies recorded the hours of brake usage until the brakes failed. The data is as follows:

- A: 998, 1023, 974, 911, 928, 981, 974, 1108, 1064, 959;
- B: 1005, 1000, 1001, 979, 1004, 1014, 1003, 966.

You are the purchasing agent for your farm. With consistency as your No. 1 priority, from which company would you buy brakes for your tractors and why? Demonstrate your reasoning with an appropriate graph or table.

**Model Assessment Answer(s):**

A graph (such as a box plot) can be constructed and the standard deviations calculated. It can be suggested that the purchasing agent recognize the differences in variation. The purchasing agent could show the results to her/his purchasing team, but further compare the results:

- Company A's standard deviation is 59.7; its mean is 991.9.
- Company B's standard deviation is 15.7; its mean is 996.4.

Company B shows more consistency because of its small standard deviation.
**COMPETENCY 9**

*Use the normal distribution to solve for the probability of an event or solve for the boundaries for a particular event.*

**Prior Knowledge:**
- Translate probabilities to percentages and percentages to probabilities.
- Solve equations involving rational numbers.

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**Real World Application Reference:**
- Agriculture and Natural Resources
- Education, Child Development, and Family Services

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**Model Assessments**

**Assessment Example 1:**

Students living away from home for the first time often face many stresses and, therefore, change their dietary and exercise habits. It is reported that freshmen living away from home for the first time and eating a typical college diet usually gain weight with a mean of 15 pounds and a standard deviation of 10 pounds. Assume the change in student weight is normally distributed.

a) Find the approximate probability that a given student will not gain weight.

b) In what range do the middle 95% of all weight changes lie?

c) For 25 randomly selected students, find the probability that the change in student weight will have a mean greater than 10 pounds.

**Model Assessment Answer(s):**

a) We want to find the probability that a student does not gain weight.

\[ P(z < -1.50) = 0.0668. \]

6.68% of all typical freshman college students do not gain weight.
b) The middle 95% fall within 2 standard deviations of the mean, so $\mu \pm 2\sigma = 15 \pm 2 \cdot 10 = -5$ and 35.

The middle 95% of all weight changes are from losing 5 pounds to gaining 35 pounds.

c) $P(x \geq 10) = P(z \geq -2.50) = 0.9938$.

The probability for 25 randomly selected students to have mean weight change greater than 10 pounds is 99.38%.

**Assessment Example 2:**

The Stanford-Binet Intelligence Scale (a type of IQ test) was originally developed to help place students in appropriate educational settings. Scores are normally and approximately distributed with a mean $\mu = 100$ and a standard deviation $\sigma = 15$.

a) What percent of students have IQ scores above 115?

b) What percent have scores below 93?

c) The Triple Nine Society is a membership association of people who have scored at the 99.9th percentile. What score on the Stanford-Binet Intelligence Scale would qualify a person for membership?
Model Assessment Answer(s):

a) \( P(x > 115) = P(z > 1.00) = 1 - 0.8413 = 0.1587, \)

so 15.87% of students have IQ scores above 115.

b) \( P(x < 93) = P(z < -0.47) = 0.3192, \)

so 31.92% of students have IQ scores below 93.

c) The z-score corresponding to the 99.9th percentile is \( z = 3.1. \)

A score of 146.5 (or 147) would qualify a person for membership in the Triple Nine Society.

Assessment Example 3:

One June day at a large nursery, the temperature was taken at 100 locations. The temperatures are normally distributed with an average temperature of 62 degrees and a standard deviation of 2 degrees. What is the probability that a random site’s temperature is more than 65 degrees?

Model Assessment Answer(s):

\( p = 0.0668. \)
COMPETENCY 10

Determine the size of a sample space using counting principles, permutations, and combinations.

Prior Knowledge:
• Apply the Fundamental Counting Principle.
• Apply the formulas for combinations and permutations.

Model Assessments

Assessment Example 1:
A daycare currently has three different activities offered in the morning, afternoon, and evening. A child can engage in one activity per time period. Because the facility is increasing in size, the director hires additional employees who help create an additional five activities per time period. The director then wishes to advertise the total number of possible arrangements of daily activities a child can experience. How many options does each child have, assuming that there is no duplication of activities?

Model Assessment Answer(s):
There are $8 \cdot 7 \cdot 6 = 336$ options.

Assessment Example 2:
a) A statistics class has 6 nursing majors, 7 environmental science majors, 35 math majors, and 1 statistics major. How many ways could a delegation of 4 students be chosen?
b) If each major has to be represented, how many ways could a delegation be chosen?

Model Assessment Answer(s):
a) 211,876.
b) 1,470.

Assessment Example 3:
You roll a die three times. How many possible outcomes can you have?

Model Assessment Answer(s):
216 different outcomes.
**COMPETENCY 11**

Compute basic probability of independent events and solve problems involving the binomial distribution:

- Determine situations that are Bernoulli trials.
- Calculate the probability of an event (success) and its complementary probability (failure).
- Discern the difference between geometric probability and binomial probability of Bernoulli trials.
- Apply the formula for binomial probability for \( x \) successes in \( n \) trials \( P(x) = \binom{n}{x} p^x q^{n-x} \).
- Use the normal model \( N(\mu, \sigma) \), where \( \mu = np \) and \( \sigma = \sqrt{npq} \) to approximate a binomial probability when the number of trials and desired successes is inordinately large.

**Prior Knowledge:**

- Apply the properties of exponents.
- Apply the formulas for combinations and permutations.
- Use notation to represent the probability of an event.

**Real World Application Reference:**

- Health Science and Medical Technology
- Manufacturing and Product Development
- Marketing, Sales, and Service

**Model Assessments**

**Assessment Example 1:**

Pop-up ads are an annoyance that most Internet surfers have encountered. An internet ad firm reports that the probability that a person clicks an ad is 0.02. Assume that clicks are independent.

a) A particular Web site with pop-up ads gets 100 visitors. What is the probability that no one will click on an ad?

b) What is the probability that five people will click on an ad?
Model Assessment Answer(s):

a) Use the binomial probability \( P(x = k) = \binom{n}{k} p^k (1-p)^{n-k} \).

\[ P(0) = \binom{100}{0} \cdot 0.02^0 \cdot 0.98^{100} = 0.1326. \]

The probability that no one will click on an ad is 13.3%.

b) Use the binomial probability \( P(x = k) = \binom{n}{k} p^k (1-p)^{n-k} \).

\[ P(5) = \binom{100}{5} \cdot 0.02^5 \cdot 0.98^{95} = 0.0353. \]

The probability that five people will click on an ad is 3.53%.

Assessment Example 2:

Given the following table from a preliminary screening test on college athletes and steroid use, find the indicated probabilities.

<table>
<thead>
<tr>
<th></th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used steroids</td>
<td>0.998</td>
<td>0.002</td>
</tr>
<tr>
<td>Did not use steroids</td>
<td>0.004</td>
<td>0.996</td>
</tr>
</tbody>
</table>

Suppose that 1.1% of a large college population has used steroids in the past.

a) What is the probability of selecting an athlete from this population who has
   i. used steroids and tested positive
   ii. used steroids and tested negative
   iii. not used steroids and tested positive
   iv. not used steroids and tested negative

b) What is the probability that the test is positive for steroids for a randomly chosen athlete from this population?

c) What is the probability that an athlete has taken steroids, given that the test is positive for steroids?
**Model Assessment Answer(s):**

a) The probability of selecting an athlete from this population who has:
   
i. used steroids and tested positive is 0.010978
   
ii. used steroids and tested negative is 0.00002
   
iii. not used steroids and tested positive is 0.003956
   
iv. not used steroids and tested negative is 0.985044

b) \( P(\text{tested positive}) = P(\text{used and tested positive}) + P(\text{did not use and tested positive}) = (0.011 \times 0.998) + (0.989 \times 0.004) = 0.010978 + 0.003956 = 0.014934. \)

c) \[ P(\text{used and tested positive}) = \frac{P(\text{used and tested positive})}{P(\text{tested positive})} \]
\[ = \frac{0.010978}{0.014934} \]
\[ = 0.74. \]

**Assessment Example 3:**

Tests by a national laboratory show that the quality of widgets from a large manufacturer has improved from 30% defective widgets in 2006 to 13% in 2010. Given that the probability that any randomly selected widget will be defective is 13%, what is the probability that a sample of 6 widgets will NOT contain a defective widget?

**Model Assessment Answer(s):**

0.4336, or 43.36%.

**Assessment Example 4:**

a) Determine if the following situations describe a Bernoulli trial:
   
i. Pick 15 cards from a deck of 52 cards without replacement. Let \( x \) be the number of black cards picked.
   
ii. Pick a card from a deck of 52, replace the card and pick another card. Repeat this process 15 times. Let \( x \) be the number of hearts picked.

b) In question ii above, determine the probability of selecting 8 hearts if you repeat the process 15 times. In this example, what event is considered a “success” and what event is considered a “failure”? Identify the number of trials, the probability of success, and the probability of failure.
Model Assessment Answer(s):

a) Do the above situations describe a Bernoulli trial?
   i. In this example there is sampling without replacement, so the selections are dependent. This is not a Bernoulli trial. Note that the probability of the first card being black is 0.5, but it keeps changing with each card that is selected.
   ii. This example is a Bernoulli trial because the number of trials is 15, $p = 0.25$, and each trial is independent of the others.

b) A success is selecting a heart; a failure is selecting any card that is not a heart, i.e., clubs, spades, or diamonds. The number of independent trials is 15; the probability of success is 0.25 and the probability of failure is 1 - 0.25, or 0.75.

$$P(8 	ext{ hearts}) = \binom{15}{8} \cdot 0.25^8 \cdot 0.75^7 = 0.0131.$$ 

The probability of selecting 8 hearts in 15 trials is 1.31%.

Assessment Example 5:

a) Suppose in a game you roll a single die until you get a 6. What is the probability that it takes four rolls before you roll a 6?

b) What is the probability that you will roll a 6 within the first four rolls?

c) Suppose in a different game you roll a single die 10 times and count the number of times you roll a 6. What is the probability that you will roll exactly 4 sixes?

Model Assessment Answer(s):

a) The first question is a Bernoulli trial with a geometric distribution.

$$P(x = 4) = \left(\frac{5}{6}\right)^{4-1} \left(\frac{1}{6}\right) = 0.096, \text{ or } 9.6\%.$$ 

b) The probability that you see a 6 within the first four rolls is:

$$\left(\frac{5}{6}\right)^0 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^1 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) = 0.518, \text{ or } 5.2\%.$$ 

c) This example is a Bernoulli trial with a binomial distribution, where $n = 10$, $x = 4$,

$$p = \frac{1}{6} \text{ and } q = (1-p) = \frac{5}{6}.$$ 

Solution: 0.054, or 5.4%.
Assessment Example 6:
A company manufactures 250,000 packages of jelly beans each week. On average, 3% of the packages do not seal properly. A random sample of 400 packages is selected at the end of the week. What is the probability that in this sample between 8 and 15 packages are not properly sealed?

Model Assessment Answer(s):
It is appropriate to use a Normal model to approximate the binomial distribution because the sample size \( n = 400 \) is very large. In this case, \( p = 0.03 \), \( q = (1 - p) = 0.97 \), \( np = 12 \), \( \sigma = \sqrt{npq} = 3.41 \). Correct for continuity by subtracting 0.5 from the lower limit of 8 and adding 0.5 to the upper limit of 15.
Solution: 0.754, or 75.4%.

Assessment Example 7:
An airline has a policy of booking as many as 15 people on an airplane that can seat only 14. The airline has determined that 85% of the booked passengers actually arrive for the flight. Find the probability that if the airline books 15 passengers, there won’t be enough seats.

Model Assessment Answer(s):
\( n = 15, \ p = 0.85, \ x = 15 \).
If all the booked persons arrive, there won’t be enough seats, thus \( x = 15 \) in this case.
\( p(15) = 0.0873 \), so there is an 8.73% chance that there won’t be enough seats.

Assessment Example 8:
A department store has experienced a 3.2% rate of customer complaints and wants to lower this rate with an employee training program. After the program, 850 customers are surveyed and 7 filed complaints.

a) Find the mean and standard deviation for the number of complaints in 850 customers.

b) Is it unusual to have 7 complaints out of 850 customers? Was the training program effective?

Model Assessment Answer(s):

a) \( \mu = np = 850(0.032) = 27.2, \)
\( \sigma = \sqrt{npq} = \sqrt{850(0.032)(0.968)} = 5.13. \)

b) The minimum usual value is 2 standard deviations below the mean:
\( \mu - 2\sigma = 27.2 - 2(5.13) = 16.94. \)
Since the seven complaints is less than 2 standard deviations below the mean, seven complaints out of 850 customers is unusually low. Thus the training was effective.
COMPETENCY 12

Apply the Central Limit Theorem to sampling distributions.

Prior Knowledge:
- Identify the sample mean and the sample standard deviation.
- Identify the population mean and the population standard deviation.

Model Assessments

Assessment Example 1:
A bank wants to apply the Central Limit Theorem to estimate the total income from their ATM charges. They decide to take a simple random sample. Which of the following statements are true according to the Central Limit Theorem? Select all that apply.

a) An increase in sample size from \( n = 16 \) to \( n = 25 \) will produce a sampling distribution with a smaller standard deviation.

b) The mean of a sampling distribution of sample means is equal to the population mean divided by the square root of the sample size.

c) The larger the sample size, the more the sampling distribution of sample means resembles the shape of the population.

d) The mean of the sampling distribution of sample means for samples of size \( n = 15 \) will be the same as the mean of the sampling distribution for samples of size \( n = 100 \).

e) The larger the sample size, the more the sampling distribution of sample means will resemble a normal distribution.

Model Assessment Answer(s):

a) The standard deviation (SD) of the sampling distribution of sample means is the standard deviation of the population SD divided by the square root of the sample size. So when sample sizes increase, the SD will decrease.

d) The mean of the sampling distribution of sample means is the mean of the population (regardless of the sample size).

e) This is by the definition of the Central Limit Theorem.

Real World Application Reference:
- Fashion and Interior Design
- Finance and Business
- Manufacturing and Product Development
- Marketing, Sales, and Service
Assessment Example 2:

A designer of exercise equipment for women needs to estimate the mean height of women to manufacture a new piece of equipment. The height of American adult women is distributed almost exactly as a normal distribution, with a mean of 63.5 inches and a standard deviation of 2.5 inches. Imagine that all possible random samples of size 25 are taken from the population of American adult women's heights. The means from each sample would then be graphed to form the sampling distribution of sample means. Draw and label the sampling distribution. Indicate the mean and standard deviation.

Model Assessment Answer(s):

Population distribution of American adult women's heights; mean is 63.5 inches and SD is 2.5 inches.

Sampling distribution of sample means from a random sample of 25 adult American women's heights; mean is 63.5 inches and SD is 0.5 inches.
Assessment Example 3:

At a local retail store, there has been an increase in complaints about employees' inattention to customer service due to the use of electronic devices. The number of personal text messages received in the last 3 minutes by employees was three, two, and five. Assume that samples of size two are randomly selected, with replacement from this population of three values. List the different possible samples and find the mean of each of them. Describe the mean and standard deviation of sampling distribution of sample means.

Model Assessment Answer(s):

The possible samples are 3-3; 3-2; 3-5; 2-3; 2-2; 2-5; 5-3; 5-2; 5-5. Their corresponding means are 3, 2.5, 4, 2.5, 2, 3.5, 4, 3.5, 5, giving a distribution of

<table>
<thead>
<tr>
<th>Mean</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1/9</td>
</tr>
<tr>
<td>2.5</td>
<td>2/9</td>
</tr>
<tr>
<td>3.0</td>
<td>1/9</td>
</tr>
<tr>
<td>3.5</td>
<td>2/9</td>
</tr>
<tr>
<td>4.0</td>
<td>2/9</td>
</tr>
<tr>
<td>5.0</td>
<td>1/9</td>
</tr>
</tbody>
</table>

with a mean of 3.33 and a standard deviation of 0.88.
COMPETENCY 13

Estimate population parameters using confidence intervals for means and proportions.

Prior Knowledge:
- Identify the sample mean and the sample standard deviation.
- Identify the population mean and the population standard deviation.

Model Assessments

Assessment Example 1:
A boat manufacturer has a problem with fiberglass cracking when its boats encounter rough waters. They try a new material and test 50 boats. The result is that 5 boats crack.

a) What is the 95% confidence interval estimating the true proportion of boats that will crack?

b) Boats using the current material crack 15% of the time. Is this new material better?

Model Assessment Answer(s):

a) \((0.01685, 0.18315)\). We are 95% confident that the interval from 0.01685 to 0.18315 contains the true proportion of boats that will crack.

b) No, the new material is not better because the current material cracks 15% of the time, which is between 1.7% and 18.3%.

Assessment Example 2:
To perform a \(z\)-confidence interval, what three simple conditions must be met?

Model Assessment Answer(s):

Simple Random Sample (SRS), normally distributed, known population standard deviation.

Assessment Example 3:
For a statistics project at your high school, you record the number of hours of sleep that 10 random students get on a given night: 7.6, 6.7, 7.5, 6.7, 7.1, 6.5, 7.7, 6.7, 7.5, 7.0.

Find the 95% confidence interval for mean amounts of sleep the students at your high school get per night. Interpret this interval in the context of the problem.

Model Assessment Answer(s):
We are 95% confident that the interval between 6.78 and 7.42 hours contains the true amount of sleep a high school student receives.

Real World Application Reference:
- Hospitality, Tourism, and Recreation
- Manufacturing and Product Development
COMPETENCY 14

Perform appropriate hypothesis test (state the hypotheses, determine the significance level, check conditions/criteria, calculate the test statistic, determine the p-value, make a decision and interpret it in the context of the problem) for a given situation.

Prior Knowledge:

• Calculate a proportion given percentages and values.
• Calculate normalized values ($z$- or $t$-values).
• Calculate the probability of a particular event using a normalized distribution.

Model Assessments

Assessment Example 1:

A soft drink company comes out with a new flavor. They give their original soft drink to a group of 23 consumers and the new soft drink to another group of 23 consumers. They then ask each group to rate how much they like the soft drink on a scale of 1-10. Determine the appropriate hypothesis test.

Model Assessment Answer(s):

Two-sample $t$-test.
Assessment Example 2:

An experiment was performed on the side effects of a new cosmetic surgery procedure. Of the 445 patients who underwent the procedure, 25 suffered an “adverse symptom.” Does this experiment provide strong evidence that fewer than 10% of patients who underwent this procedure have adverse symptoms? Perform an appropriate test at the $\alpha = 0.01$ significance level.

Model Assessment Answer(s):

**Hypotheses:** We want to test a claim about $p$, the true proportion of patients who suffered an adverse symptom. Our hypotheses are:

- $H_0: p = 0.10$
- $H_a: p < 0.10$

**Conditions:** We should use a one-proportion z-test if the conditions are met.

- **SRS:** We are assuming that a “random sample” of patients undergoing a new cosmetic surgery procedure was analyzed. If the sampling design was not an SRS, our calculations may not be accurate.
- **Normality:** The expected number of “yes” and “no” responses are $(445)(0.10) = 44.5$ and $(445)(0.90) = 400.5$, respectively. Both are at least 10, so we can use the z-test.
- **Independence:** Since we are sampling without replacement, this hospital must have at least $(10)(445) = 4,450$ patients undergoing the new procedure.

**Test-statistic:** The one-proportion z-statistic is

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.0562 - 0.10}{\sqrt{\frac{0.0562(0.9488)}{445}}} = -3.02.$$

**P-value:** $P(z \leq -3.02) = 0.0010$.

**Interpretation:** There is a 0.10% chance of obtaining a sample result as unusual as or even more unusual than we did ($\hat{p} = 0.0562$) when the null hypothesis is true. Since our $p$-value of 0.0010 is less than $\alpha = 0.01$, reject $H_0$. We have significant evidence to say that the proportion of patients undergoing this new cosmetic surgery procedure who suffer an adverse symptom is less than 10%.
**Assessment Example 3:**

A school newspaper reported on a study of bacteria found on teachers’ keyboards in a high school. A total of 50 keyboards were sampled and analyzed for bacteria. The bacterial concentration of each specimen was determined using an infrared microscopic method. The sample resulted in a mean bacterial level of 84 ug/mL and a standard deviation of 80 ug/mL. Test the hypothesis that the true mean bacteria level in teachers’ keyboards falls below 100 ug/mL. Use $\alpha = 0.01$.

**Model Assessment Answer(s):**

**Hypotheses:** We want to test a claim about the true population mean bacteria level found on teachers’ keyboards. Our hypotheses are

\[
H_0: \mu = 100, \\
H_a: \mu < 100.
\]

**Conditions:** Since we do not know the standard deviation of the bacteria level found on all teachers’ keyboards, we must use a one-sample $t$-test.

- **SRS:** We must be willing to treat our 50 keyboards as an SRS from the population of all teachers’ keyboards if we want to draw conclusions about teachers’ keyboards in general. This is a judgment call.

- **Normality:** Use of the $t$-procedure is justified because the sample size is reasonably large ($n = 50$) and thus the distribution of $\bar{x}$ will be approximately Normal by the Central Limit Theorem.

- **Independence:** We are sampling without replacement and the population size is much larger than $10 \times 50 = 500$.

**Test statistic:** The $t$-test statistic is

\[
t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{84 - 100}{80 / \sqrt{50}} = -1.412.
\]

**P-value:** Although the degrees of freedom are $df = n - 1$, so $df = 50 - 1 = 49$, we use the more conservative $df = 40$ from a standard $t$-table. For $t = -1.412$, the upper tail probability is between 0.10 and 0.05. Using the calculator, $p = 0.0818$.

**Interpretation:** Since our $p$-value (0.0818) is not less than $\alpha = 0.01$, we fail to reject $H_0$. We do not have sufficient evidence to say that the mean level of bacteria found on teachers’ keyboards is less than 100 ug/mL.
**Assessment Example 4:**

According to a report, the rate of firearm deaths per 100,000 people for the United States was 10.34. The rate of firearm deaths per 100,000 for Kansas was 10.51. Test the hypothesis that the rate of firearm deaths per 100,000 in Kansas is significantly different from the rate for the United States. Use \( \alpha = 0.05 \).

**Model Assessment Answer(s):**

- **Hypotheses:** We want to test a claim about \( p_1 \), the true proportion of firearm deaths in Kansas, and \( p_2 \), the true proportion of firearm deaths in the United States. Our hypotheses are
  \[ H_0: p_1 = p_2, \quad H_a: p_1 \neq p_2. \]

- **Test-statistic:** The two-proportion \( z \)-statistic is \( z = -0.37 \).

- **P-value:** \( p = 0.7097 \).

- **Interpretation:** There is a 71.0% chance of obtaining a sample result as unusual as or even more unusual than we did, so we do not have enough evidence to say that the firearm death rates are different.

---

**Assessment Example 5:**

A local municipal airport conducted a study over several years of pilot errors and the number of DWIs (driving while intoxicated). The following table shows the data collected.

<table>
<thead>
<tr>
<th></th>
<th>No DWIs</th>
<th>1 or more DWIs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pilots with no</td>
<td>21,588</td>
<td>608</td>
</tr>
<tr>
<td>pilot-error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>accidents</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pilots with pilot-</td>
<td>199</td>
<td>11</td>
</tr>
<tr>
<td>error accidents</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test the hypothesis that there is no relationship between the number of DWIs and the number of pilot-error accidents. Use \( \alpha = 0.05 \).

**Model Assessment Answer(s):**

- **Hypotheses:**
  - \( H_0: \) There is no association between the number of DWIs and the number of pilot-error accidents.
  - \( H_1: \) There is an association between the number of DWIs and the number of pilot-error accidents.

- **Test-statistic:** \( \chi^2 = 4.836 \).

- **P-value:** \( p = 0.0279 \).

- **Interpretation:** There is a 2.79% chance of obtaining a sample result as unusual as or even more unusual than we did, so we reject the null hypothesis. The evidence appears to suggest that there is an association between the number of DWIs and the number of pilot-error accidents.
Assessment Example 6:

Suppose that four cough syrups are compared in an experiment, with the hours of relief from coughing as the measured variable. Do these cough syrups have the same mean hours of relief?

<table>
<thead>
<tr>
<th>Hours of relief</th>
<th>Cough Syrup A</th>
<th>Cough Syrup B</th>
<th>Cough Syrup C</th>
<th>Cough Syrup D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.53</td>
<td>9.80</td>
<td>5.79</td>
<td>19.19</td>
</tr>
<tr>
<td></td>
<td>0.77</td>
<td>1.90</td>
<td>1.91</td>
<td>7.17</td>
</tr>
<tr>
<td></td>
<td>12.99</td>
<td>11.45</td>
<td>1.97</td>
<td>8.96</td>
</tr>
<tr>
<td></td>
<td>7.41</td>
<td>4.97</td>
<td>8.73</td>
<td>5.28</td>
</tr>
<tr>
<td></td>
<td>8.82</td>
<td>1.76</td>
<td>11.13</td>
<td>17.18</td>
</tr>
<tr>
<td></td>
<td>4.40</td>
<td>11.18</td>
<td>13.01</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>16.35</td>
<td>9.49</td>
<td>3.86</td>
<td>4.23</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>17.55</td>
<td>8.77</td>
<td>10.85</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>2.34</td>
<td>-</td>
<td>6.73</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>12.99</td>
<td>-</td>
<td>2.49</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>17.86</td>
<td>-</td>
<td>9.06</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9.99</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.48</td>
</tr>
</tbody>
</table>

**Model Assessment Answer(s):**

\( H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4. \)

\( H_1: \) The population means are not all equal.

**Test statistic:** \( F = 0.3269. \)

**P-value:** \( p = 0.8059. \)

We fail to reject the null hypothesis. The mean time of relief for the four syrups do not appear to be statistically different.
COMPETENCY 15

Explain the relationship between parameters and statistics.

Prior Knowledge:

• Use basic statistics terminology.
• Calculate the mean and standard deviation for a given data set.
• Identify the population and sample in a given scenario.

Model Assessments

Assessment Example 1:
A novice businessperson has a business idea and is interested in obtaining a loan. A friend who works at a bank tells her that the average business loan rate in this area is 8.25%. She does some research and finds five banks that give an average rate of 10.25%. Determine which percentage is a parameter and which is a statistic.

Model Assessment Answer(s):
8.25% is a parameter and 10.25% is a statistic.

Assessment Example 2:
The public health agency worker has been concerned about the ever-increasing food contamination issues in her city. After inspecting 55,000 kilograms of meat stored at a local sausage company, she found that 45,000 kilograms of the meat was spoiled. Identify the sample, population, statistic, and parameter (if present).

Model Assessment Answer(s):
The sample is 55,000 kilograms of meat stored at the sausage company, and the statistic is the 45,000 kilograms of spoiled meat. In this question, the population is considered to be all possible meat produced in this city, and the parameter is the total kilograms of spoiled meat from the city.
Assessment Example 3:
A health and fitness club manager surveys 40 randomly selected members regarding the ideal dimensions of the new swimming pool. He found that the average ideal dimensions of those questioned to be 100 yards by 25 yards. Identify the sample, population, statistic, and parameter (if present).

Model Assessment Answer(s):
The sample is 40 randomly selected members and the statistic is the average ideal dimensions of 100 yards by 25 yards. The population is considered to be all possible members of this club, and the parameter is the mean of all ideal dimensions from all club members.

Assessment Example 4:
A poll was done for an election and a random selection of 1,000 citizens was asked if they were in favor of a national universal health care bill. Of those polled, 175 citizens indicated that they believed the health care bill was a necessity. Identify the sample, population, statistic, and parameter (if present).

Model Assessment Answer(s):
The sample is the 1,000 citizens randomly selected and the statistic is the proportion (0.175) of citizens in favor of the bill. The population is considered to be all citizens of this nation, and the parameter is the proportion of all the citizens of this nation in favor of the bill.
COMPETENCY 16

Given a p-value, interpret its meaning in the context of the variables of a problem:

- Define the p-value.
- Identify the appropriate null and alternative hypotheses, and use the proper notation of $H_0$ and $H_a$.
- Interpret a given p-value in the context of a hypothesis testing situation as it relates to the rejection of or failure to reject $H_0$.

Prior Knowledge:

- Identify the probability of an event using a standardized normal distribution.
- Identify the parameter and statistic for a given scenario.
- Calculate $z$- and $t$-values for a given scenario.

Model Assessments

Assessment Example 1:

A tourism service wishes to determine if more people contact them the week before New Year’s Day or the week before Labor Day. A student intern gathers data from the previous 35 months and determines the sample average number of people contacting them a week before each holiday. They then run a two-sample $t$-test using the sample averages. They obtain a $p$-value of 0.1325. What can be concluded?

Model Assessment Answer(s):

There is no statistical difference between the average number of people who contact the tourism service the week before New Year’s Day and the week before Labor Day.
**Assessment Example 2:**

A fashion merchandiser wants to know if increasing the price on their most purchased dress increased the total sales. They randomly choose 30 stores and perform a matched pair comparison, comparing the average total sales of dresses before and after the price increase. They obtain a \( p \)-value of \( 0.003556 \). What can be concluded?

**Model Assessment Answer(s):**

The fashion merchandiser should increase the price in all of her/his stores. There is a statistically significant increase in the average total sales of dresses after the price increase.

**Assessment Example 3:**

A new drug is being tested by the FDA that reduces the duration of migraines to less than 2 hours. This observation has a \( p \)-value = 0.032. Interpret the meaning of the \( p \)-value in the context of the problem.

**Model Assessment Answer(s):**

The \( p \)-value = 0.032 indicates that the new drug, indeed, has an effect on reducing the duration of migraines to less than 2 hours. There is a statistically significant reduction in the duration of migraines.
COMPETENCY 17

Find and apply the equation of the regression line when linear regression is appropriate:

- Determine when linear regression is appropriate for a set of bivariate data.
- Find the linear correlation coefficient \( r \) and the coefficient of determination \( r^2 \).
- Use the regression line to make appropriate predictions.
- Interpret \( r \), \( r^2 \), and slope in the context of a given situation.

Prior Knowledge:

- Graph linear equations.
- Find the slope and \( y \)-intercept of a line.

Real World Application Reference:

- Education, Child Development, and Family Services
- Energy and Utilities
- Finance and Business
- Health Science and Medical Technology
- Information Technology

Model Assessments

Assessment Example 1:

Your friend recently became the new manager of a pizza restaurant. He wants to determine if the current prices for cheese pizzas are consistent with their size. Prices for cheese pizza are shown in the table below.

<table>
<thead>
<tr>
<th>Size</th>
<th>Diameter (inches)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal</td>
<td>6.5</td>
<td>$4.69</td>
</tr>
<tr>
<td>Small</td>
<td>9.5</td>
<td>$9.00</td>
</tr>
<tr>
<td>Medium</td>
<td>12.0</td>
<td>$13.25</td>
</tr>
<tr>
<td>Large</td>
<td>14.0</td>
<td>$15.99</td>
</tr>
<tr>
<td>Extra Large</td>
<td>15.0</td>
<td>$20.25</td>
</tr>
</tbody>
</table>

a) Make a scatter plot of these data.
b) Explain how you would determine if there is a linear relationship between the diameter and cost.
Model Assessment Answer(s):

a) scatter plot

![Graph showing scatter plot with points and line of best fit.]

b) There is a curve in the residual plot, so a linear relationship is not appropriate. A power model would be better. According to the new power model \( \hat{y} = (10)^{-0.697} \cdot (x)^{1.684} \), the predicted cost of the five different sizes of pizza are $4.70, $8.91, $13.20, $17.12, and $19.22. It appears that a slight adjustment in the charges for large and extra large pizzas might be warranted.

Assessment Example 2:

Typically, the temperature in Phoenix, AZ, is high in July. The temperature is more moderate in January. Planners at the Phoenix Electric Company would like to use January's average temperatures to predict July's average temperature. The following table shows the average temperatures in Fahrenheit for January and July in Phoenix from 2001 to 2009 (data available at www.noaa.gov).

<table>
<thead>
<tr>
<th>Year</th>
<th>January temperatures in degrees Fahrenheit</th>
<th>July temperatures in degrees Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>54.1</td>
<td>94.4</td>
</tr>
<tr>
<td>2002</td>
<td>56.0</td>
<td>96.0</td>
</tr>
<tr>
<td>2003</td>
<td>62.0</td>
<td>97.7</td>
</tr>
<tr>
<td>2004</td>
<td>57.5</td>
<td>94.5</td>
</tr>
<tr>
<td>2005</td>
<td>57.8</td>
<td>97.3</td>
</tr>
<tr>
<td>2006</td>
<td>57.7</td>
<td>96.5</td>
</tr>
<tr>
<td>2007</td>
<td>52.9</td>
<td>95.8</td>
</tr>
<tr>
<td>2008</td>
<td>54.7</td>
<td>94.9</td>
</tr>
<tr>
<td>2009</td>
<td>58.7</td>
<td>98.3</td>
</tr>
</tbody>
</table>
a) Create a scatter plot with the January temperatures on the x-axis and the July temperatures on the y-axis.

b) Find the equation of the least-squares regression line.

c) What is the correlation between July weather and January weather?

**Model Assessment Answer(s):**

a) scatter plot

![Scatter plot](image)

b) $\hat{y} = 76.159 + 0.352x$.

c) $r = 0.685$. (NOTE: The correlation is statistically significant at $\alpha = 0.05$, but it is not at $\alpha = 0.01$.)

**Assessment Example 3:**

The winning times (in seconds) in the men’s 100-meter dash in the Olympics from 1972 to 2008 are given in the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Athlete</th>
<th>Country</th>
<th>Winning times in seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>Valery Borzov</td>
<td>URS</td>
<td>10.14</td>
</tr>
<tr>
<td>1976</td>
<td>Hasely Crawford</td>
<td>TRI</td>
<td>10.06</td>
</tr>
<tr>
<td>1980</td>
<td>Allan Wells</td>
<td>GBR</td>
<td>10.25</td>
</tr>
<tr>
<td>1984</td>
<td>Carl Lewis</td>
<td>USA</td>
<td>9.99</td>
</tr>
<tr>
<td>1988</td>
<td>Carl Lewis</td>
<td>USA</td>
<td>9.92</td>
</tr>
<tr>
<td>1992</td>
<td>Linford Christie</td>
<td>GBR</td>
<td>9.96</td>
</tr>
<tr>
<td>1996</td>
<td>Donovan Bailey</td>
<td>CAN</td>
<td>9.84</td>
</tr>
<tr>
<td>2000</td>
<td>Maurice Greene</td>
<td>USA</td>
<td>9.87</td>
</tr>
<tr>
<td>2004</td>
<td>Justin Gatlin</td>
<td>USA</td>
<td>9.85</td>
</tr>
<tr>
<td>2008</td>
<td>Usain Bolt</td>
<td>JAM</td>
<td>9.69</td>
</tr>
</tbody>
</table>
a) Make a scatter plot of the data where \( x \) represents the number of years since 1972 and \( y \) represents the winning times.

b) Find the equation for the least squares line for this data.

c) Interpret the slope in context of the question.

d) How can you determine if the least squares line is an appropriate model for the data?

**Model Assessment Answer(s):**

a) scatter plot

![Scatter plot](image)

b) \( \hat{y} = 10.1705 - 0.0119x \).

c) For each increase of one year, the Olympic record decreases by 0.0119 seconds.

d) A residual plot reveals a random scattering of points. The residuals vary from −0.06 to 0.17, so a least squares line is an appropriate model.
Assessment Example 4:

Since bone structure varies in size from person to person, researchers have added frame size as a factor in helping to determine a person’s ideal weight. Knowing one’s frame size can help a person set reasonable weight goals.

The students in a statistics class measured their wrist circumference and height in centimeters. The following data were found.

<table>
<thead>
<tr>
<th>Name</th>
<th>Wrist circumference (in cm.)</th>
<th>Height (in cm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karen</td>
<td>15.0</td>
<td>163</td>
</tr>
<tr>
<td>Chris</td>
<td>17.0</td>
<td>191</td>
</tr>
<tr>
<td>Colleen</td>
<td>15.5</td>
<td>168</td>
</tr>
<tr>
<td>Cayley</td>
<td>15.5</td>
<td>173</td>
</tr>
<tr>
<td>Ethan</td>
<td>16.0</td>
<td>183</td>
</tr>
<tr>
<td>Alex</td>
<td>16.5</td>
<td>175</td>
</tr>
<tr>
<td>Krizza</td>
<td>14.5</td>
<td>156</td>
</tr>
<tr>
<td>Julia</td>
<td>16.0</td>
<td>172</td>
</tr>
<tr>
<td>Carrie</td>
<td>17.5</td>
<td>176</td>
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<tr>
<td>Gissel</td>
<td>17.0</td>
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<tr>
<td>Natalie</td>
<td>15.5</td>
<td>162</td>
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<tr>
<td>Jeff</td>
<td>17.0</td>
<td>188</td>
</tr>
<tr>
<td>Ms. Statsky</td>
<td>14.0</td>
<td>168</td>
</tr>
</tbody>
</table>

a) Make a scatter plot of the data. Does the relationship appear to be approximately linear?

b) Determine the equation of the least squares regression line.

c) Compute the residuals and make a plot of the residuals against x. Describe the pattern displayed by the residuals.

d) What is $r^2$? Interpret this value in context. What does this value indicate about the success of the linear regression in predicting a student’s height?
Model Assessment Answer(s):

a) scatter plot

![Scatter plot of height vs. wrist circumference](image)

Yes, it looks approximately linear.

b) \( \hat{y} = 76.89 + 6.05x \).

c) scatter plot

![Residual plot](image)

They are scattered about the line \( y = 0 \), with some points far from this line. There is no distinct pattern.
d) \( r^2 = 0.3735 \), so 37.35% of the variation in height is explained by the least squares regression line of height on wrist circumference. This value is not very good; from the table we can see that this regression line was off by 15.7 centimeters for one student.

<table>
<thead>
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<tr>
<td>-3.573</td>
</tr>
<tr>
<td>12.335</td>
</tr>
<tr>
<td>-1.596</td>
</tr>
<tr>
<td>3.404</td>
</tr>
<tr>
<td>10.381</td>
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<tr>
<td>-0.642</td>
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<tr>
<td>-7.55</td>
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<tr>
<td>-0.619</td>
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<tr>
<td>-5.688</td>
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<td>-15.66</td>
</tr>
<tr>
<td>-7.596</td>
</tr>
<tr>
<td>9.335</td>
</tr>
<tr>
<td>7.473</td>
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Assessment Example 5:
Twenty students are randomly selected from a college statistics class of 80 students. Their most recent homework and exam scores are compared.

<table>
<thead>
<tr>
<th>Homework scores (points out of 10 possible points)</th>
<th>Exam scores (points out of 100 possible points)</th>
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<td>10.0</td>
<td>79.0</td>
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<tr>
<td>7.5</td>
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<td>78.0</td>
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<td>8.5</td>
<td>89.5</td>
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</table>
a) Identify the explanatory and response variables.

b) Create a scatter plot and find the equation for the least squares line.

c) State the slope of the least squares regression line and interpret the meaning of the slope in terms of the predicted response variable and the explanatory variable.

d) If a student scores a 7 on homework, what is the predicted exam score for that student?

**Model Assessment Answer(s):**

a) The explanatory variable is homework scores and the response variable is exam scores.

b) scatter plot

![Figure S17-5b](50 55 60 65 70 75 80 85 90 95 5 6 7 8 9 10 11)

Predicted exam score = 66.73 + 1.16 • homework score.

c) The slope is 1.1615, and it can be interpreted in the following way: For every 1 point gain in homework score, the predicted exam score is 1.1615 points higher on average.

d) The predicted exam score is 77.0. Since $r = 0.127$, the linear correlation is not statistically significant. Hence, the best predicted exam score is the average exam score, 76.95.

**Assessment Example 6:**

Solar panels improve energy efficiency. If they become dirty, efficiency can drop by 10–15%. Hamat wants to see if there is a relationship between the number of times he cleans his solar panels each year and the total power output. After performing a linear regression on the data, he obtains a correlation coefficient of 0.345. Interpret this solution in the context of the situation.

**Model Assessment Answer(s):**

The $r$ value is low, suggesting that there is a weak linear relationship between the number of times he cleans his solar panels each year and the total power output.
Assessment Example 7:
The director of a quality department in a network communications company wants to know how many bugs each person fixes per day. His network does not inform him of the number of bugs each person fixes, but rather the total number of bugs fixed per day of all employees. However, he does have a history of data and is able to plot the total number of bugs fixed per day for different total numbers of employees. The graph has a slope of 1.32, a correlation coefficient of 0.9, and a coefficient of determination of 0.81. Interpret each of these numbers.

Model Assessment Answer(s):
For each additional employee, 1.32 more bugs are fixed on average per day. The very high \( r \) value suggests that the relationship is strongly linear. The \( r^2 \) value states that 81% of the variability in the bugs fixed per day can be explained by the number of employees.

Assessment Example 8:
Listed below are the budgets (in millions of dollars) and the gross receipts (in millions of dollars) for randomly selected movies.

<table>
<thead>
<tr>
<th>Budget</th>
<th>62</th>
<th>90</th>
<th>50</th>
<th>35</th>
<th>200</th>
<th>100</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross receipts</td>
<td>65</td>
<td>64</td>
<td>48</td>
<td>57</td>
<td>601</td>
<td>146</td>
<td>47</td>
</tr>
</tbody>
</table>

a) Make a scatter plot of this data.

b) Is there a linear relationship between budget and gross receipts?

Model Assessment Answer(s):

a) scatter plot

b) Even though the correlation coefficient is \( r = 0.926 \), which is indicative of a strong linear relationship, the plot of the residuals is not random. This indicates that there is not a linear relationship between budget and gross receipts. Even though there is not a linear relationship, there is some other kind of relationship (exponential, quadratic, or other) as shown by the data.
ACKNOWLEDGEMENTS/CONTRIBUTORS

Math faculty from 13 Cal-PASS Professional Learning Councils collaborated toward the common goal of aligning curricula from high school through transfer-level coursework in postsecondary education in the mathematics subject areas of Algebra II/Intermediate Algebra, Precalculus, and (non-STEM) Statistics. The 13 Cal-PASS math Professional Learning Councils that participated in the ACCESS initiative were from the following regions:

Contra Costa
East San Diego County
Los Angeles
Merced
North Bay
North Coast
Placer-Nevada
San Francisco
San Mateo
Santa Barbara
Siskiyou County
West Fresno Region
West San Bernardino County

Several Cal-PASS Professional Learning Councils began this type of alignment work in their local regions prior to the ACCESS Initiative, and their experience helped Cal-PASS develop a model for aligning exit and entrance competencies for sequential coursework. This initiative is unique in that it brought together a collaboration of high school and postsecondary faculty from around the state to discuss curriculum and build a clearly articulated curricula guide chronicling what students are expected to know upon completing one course and to be prepared for subsequent courses. The names of Cal-PASS Professional Learning Council faculty participants who graciously volunteered their time and expertise for this project are listed on the following pages.

The ACCESS initiative was made possible through partnerships between Cal-PASS and The William and Flora Hewlett Foundation, The James Irvine Foundation, The Evelyn and Walter Haas Jr. Fund, and The Girard Foundation. This far-reaching project could not have come to fruition without the keen vision and generous funding of these partners.
The coordination and management of this statewide endeavor was the accomplishment of Eden Dahlstrom, ACCESS project director, who traveled around the state over a 2-year period to meet with the Cal-PASS Professional Learning Councils and keep the project relevant and on track. She also headed up the summer workshops, where faculty from around the state came together to specify competencies and assessments. Cal-PASS leaders Michelle Kalina and Shelly Valdez also traveled to Cal-PASS Professional Learning Council meetings and offered collegial guidance and assistance to keep this project energized and relevant.

After hundreds of pages of math assessments and equations were compiled, the guide was handed over to math instructors Debbie Hill and Kentaro Iwasaki to handle the initial editing process. Following several rounds of edits, Cal-PASS’s Katheryn Horton stepped up to oversee the editing process. Math instructor Michael Orr took over the reins as content editor and created most graphs and figures. Finally, the math team of Michael Orr, Tina Shinsato, and Diane Murillo edited and honed the content into its final form. Throughout the editing process, freelance copyeditor Cindi Patton (www.thefinaledit.com) handled copyediting and formatting as well as production and print management. Patton Brothers Illustration & Design, Inc., (www.pattonbros.com) created the design. Math instructors Michelle McGinity, Sherry Wallin, Vanson Nguyen, and Gary Remiker also contributed to the editing and proofing of this guide.

For more information, please contact Dr. Shelly Valdez, EdD, at svaldez@iebcnow.org.
# Math Faculty Participants

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## Acknowledgements/Contributors

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<td>White, Wes</td>
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# Acknowledgements/Contributors

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<td>Whitley, Sheila</td>
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# Career and Technical Education Participants

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<tbody>
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<td>Davis, Bill</td>
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<tr>
<td>Harvey, Elizabeth</td>
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<td>Hollems, Diane</td>
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<td>Miller, Jerald</td>
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<td>White, Kathleen</td>
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# Cal-PASS Regional Coordinators

<table>
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<tr>
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<tbody>
<tr>
<td>Ceaser, Lisbeth</td>
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<td>Horton, Katheryn</td>
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<td>Iwasaki, Kentaro</td>
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<td>Linfor, Cali</td>
<td>San Diego - East</td>
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<tr>
<td>Mahar, Kate</td>
<td>San Francisco and Contra Costa</td>
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<td>Owens, Rae</td>
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<td>Schneider, Garry</td>
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<td>Schneider, Katy</td>
<td>San Bernardino - West</td>
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<td>Fresno - West</td>
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<td>Tyberg, Alana</td>
<td>San Diego - East</td>
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<tr>
<td>Wintermeyer, Lauren</td>
<td>Santa Barbara</td>
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**APPENDIX**

**Mapping ACCESS Competencies to California Content Standards and Common Core Standards**

The tables on the following pages map the relationship between ACCESS competencies and California-based and nationally based standards. The competencies are not necessarily organized in the same way as the California State Standards or Common Core Standards, nor are they necessarily in the order in which one would teach them in the course. Additionally, some of the ACCESS competencies do not have a corresponding map to California Content Standards or Common Core Standards. However, the faculty involved in this project felt that these skills were important enough that they be included as ACCESS competencies because they are necessary for the next level of coursework.

**The California State Standards** were designed by appointees of the State Board of Education. This document does not retain the official input of California’s public colleges and universities. The document was adopted by the State Board of Education and mandated for all public K-12 schools. These standards are available online at [www.cde.ca.gov/board/](http://www.cde.ca.gov/board/).

**The Common Core State Standards Initiative** is a state-led effort launched in 2009 by state leaders, including governors and state commissioners of education from 48 states through membership in the National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). The process used to write the standards was informed by:

- the best of state standards
- the experience of teachers, content experts, states, and leading thinkers
- feedback from the general public

To write the standards, the NGA Center and CCSSO brought together content experts, teachers, researchers, and others.

The Common Core standards, which can be found online at [www.corestandards.org](http://www.corestandards.org), are divided into two categories:

- college and career readiness standards, which address what students are expected to learn when they have graduated from high school
- K-12 standards, which address expectations for elementary through high school
The California Education RoundTable Content Standards (called the CERT Standards) were published just prior to the state’s content standards for mathematics in 1997. The standards were designed by a task sponsored by the RoundTable. The RoundTable comprised, in part, the heads of the California Department of Education, the California Community Colleges, the California State Universities, and the University of California. Serving on the task force were K-12 faculty, administrators, public participants, and academic Senate-appointed postsecondary faculty. While these standards have no official status under the Board of Education, they provide a point of contrast, noting competencies that are not addressed in the California mathematics standards. These standards can be found at www.certicc.org.

CERT Mathematics Standards for California High School Graduates (not included in the rubric on the following pages): This report identifies the kinds of activities that should be promoted in the classroom. We include them here in summary form:

- **Becoming Fluent in Mathematics** — Effective classes will provide experiences that lead to students’ acquisition of facility with the basic techniques of mathematics. There are certain necessary skills that students will need to be able to call upon without hesitation.
- **Modeling Mathematical Thinking** — Effective classes will reflect the enthusiasm that comes in learning with a teacher and others who get excited about mathematics, who work as a team, who experiment and form conjectures.
- **Solving Problems** — Effective classes will reflect the notion that problem solving is best conveyed by giving students appropriate experience in solving unfamiliar problems, by then engaging them in discussion of their various attempts at solutions, and by reflecting on these processes.
- **Developing Analytic Ability and Logic** — Effective teachers will emphasize a thorough understanding of the subject matter and the development of logical reasoning. A classroom full of discourse and interaction that focuses on reasoning is a classroom in which analytic ability and logic are being developed.
- **Experiencing Mathematics in Depth** — Students must delve deeply into well-chosen areas of mathematics. A shallow exploration of a broad range of topics will not contribute to the ability to understand and independently use mathematics.
- **Appreciating the Beauty and Fascination of Mathematics** — Effective teachers must nurture the appreciation for the inherent beauty of mathematics.
- **Building Confidence** — Effective teachers must create situations in which students are rewarded for being inquisitive, for experimenting, for taking risks, and for persisting in finding solutions they fully understand. This will help students generate confidence in their mathematical ability.
- **Communicating** — Effective classes will reflect a student’s ability to explain with confidence not only the answer to the problem but the process by which the problem was solved. Students need extensive experience in oral and written communication with extensive feedback in order to develop these skills.
### Mapping ACCESS Algebra II/Intermediate Algebra Competencies to Standards

<table>
<thead>
<tr>
<th>Competency Statement (Algebra II/Intermediate Algebra)</th>
<th>ACCESS Competency Number</th>
<th>CA Content Standard</th>
<th>National Common Core Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use properties of absolute values to solve multistep equations and inequalities.</td>
<td>1</td>
<td>1.0</td>
<td>A-CED 1 A-REI 3</td>
</tr>
<tr>
<td>Solve systems of linear equations in both two and three variables by substitution, elimination/addition, graphs, and matrices.</td>
<td>2</td>
<td>2.0</td>
<td>A-REI 5–9 A-CED 3 N-VM 6</td>
</tr>
<tr>
<td>Factor polynomials by grouping.</td>
<td>3</td>
<td>4.0</td>
<td>A-SSE 2</td>
</tr>
<tr>
<td>Factor polynomials by using the sum/difference of cubes pattern.</td>
<td>4</td>
<td>4.0</td>
<td>A-SSE 2</td>
</tr>
<tr>
<td>Factor polynomials by extending the difference of squares pattern to polynomials of degree higher than 2.</td>
<td>5</td>
<td>4.0</td>
<td>A-SSE 2</td>
</tr>
<tr>
<td>Solve quadratic equations in the real and complex number systems by factoring.</td>
<td>6</td>
<td>8.0</td>
<td>N-CN 7 A-SSE 3a F-IF 8a</td>
</tr>
<tr>
<td>Solve quadratic equations in the real and complex number systems by completing the square.</td>
<td>7</td>
<td>8.0</td>
<td>A-SSE 3b A-REI 4a and 4b F-IF 8a</td>
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<tr>
<td>Solve quadratic equations in the real and complex number systems by using the quadratic formula.</td>
<td>8</td>
<td>8.0</td>
<td>A-REI 4b</td>
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<tr>
<td>Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in quadratic equations.</td>
<td>9</td>
<td>8.0 and 9.0</td>
<td>A-CED 1 F-IF 7a and 8a F-LE 3</td>
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<tr>
<td>Add and subtract complex numbers.</td>
<td>10</td>
<td>6.0</td>
<td>N-CN 2</td>
</tr>
<tr>
<td>Multiply complex numbers.</td>
<td>11</td>
<td>7.0</td>
<td>N-CN 2 and 5</td>
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<tr>
<td>Competency Statement (Algebra II/Intermediate Algebra)</td>
<td>ACCESS Competency Number</td>
<td>CA Content Standard</td>
<td>National Common Core Standard</td>
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<tr>
<td>------------------------------------------------------</td>
<td>--------------------------</td>
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<tr>
<td>Perform division with complex numbers.</td>
<td>12</td>
<td>7.0</td>
<td>N-CN 3</td>
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<tr>
<td>Simplify rational expressions with higher order polynomials in the denominator by canceling common factors in the numerator and denominator.</td>
<td>13</td>
<td>3.0</td>
<td>A-APR 1, 6, and 7</td>
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<tr>
<td>Add and subtract rational expressions with higher order polynomials in the denominator.</td>
<td>14</td>
<td>7.0</td>
<td>A-APR 7</td>
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<tr>
<td>Multiply and divide rational expressions with higher order polynomials in the denominator.</td>
<td>15</td>
<td>6.0</td>
<td>A-APR 6 and 7</td>
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<tr>
<td>Divide polynomials by binomials using long division and synthetic division.</td>
<td>16</td>
<td>12.0</td>
<td>A-APR 6</td>
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<tr>
<td>Simplify a fraction where the numerator, denominator, or both contain a fraction (complex fractions).</td>
<td>17</td>
<td>12.0</td>
<td>A-APR 6</td>
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<tr>
<td>Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in equations involving rational expressions.</td>
<td>18</td>
<td>15.0</td>
<td>A-REI 2, A-APR 6</td>
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<tr>
<td>Use the properties of exponents to simplify expressions with rational exponents.</td>
<td>19</td>
<td>15.0</td>
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<tr>
<td>Use the properties of exponents to solve equations with rational exponents.</td>
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<td>15.0</td>
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<td>Simplify radical expressions by removing repeated factors and performing addition and subtraction operations.</td>
<td>21</td>
<td>1.0, 9.0, 10.0, 12.0, 13.0, 16.0, and 17.0</td>
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<td>Competency Statement (Algebra II/Intermediate Algebra)</td>
<td>ACCESS Competency Number</td>
<td>CA Content Standard</td>
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<tr>
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<tr>
<td>Simplify radical expressions by removing repeated factors and performing multiplication operations.</td>
<td>22</td>
<td>1.0, 9.0, 10.0, 12.0, 13.0, 16.0, and 17.0</td>
<td>A-REI 2</td>
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<td>Rationalize the denominator of radical expressions.</td>
<td>23</td>
<td>24.0</td>
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<td>Use multiple representations (tables, graphs, and equations) to solve problems that result in radical equations.</td>
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<td>24.0</td>
<td>No corresponding standard</td>
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<tr>
<td>Apply and graph transformations of parent functions (quadratic, cubic, square root, absolute value, exponential, and logarithmic).</td>
<td>25</td>
<td>12.0</td>
<td>F-LE 2 and 3 F-IF 7</td>
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<tr>
<td>Identify the parent function (quadratic, cubic, square root, absolute value, exponential, and logarithmic) and transformations for a given transformed function in algebraic or graphical form.</td>
<td>26</td>
<td>11.0-14.0</td>
<td>F-LE 2 F-BF 5</td>
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<tr>
<td>Compose two linear, quadratic, cubic, square root, absolute value, exponential, or logarithmic functions.</td>
<td>27</td>
<td>16.0</td>
<td>F-BF 1c</td>
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<tr>
<td>Find the inverse of linear, quadratic, cubic, square root, absolute value, exponential, and logarithmic functions algebraically and graphically.</td>
<td>28</td>
<td>18.0–20.0</td>
<td>F-BF 4 and 5</td>
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<td>Find the equation of an exponential function given two points on the graph.</td>
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<td>22.0–23.0</td>
<td>F-BF 2 F-LE 2</td>
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<td>Use multiple representations (tables, graphs, and equations) to solve contextualized problems that result in one- and two-step exponential and logarithmic equations.</td>
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<td>Linear Algebra 1-12</td>
<td>F-BF 1 and 5</td>
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<tr>
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<tr>
<td>Graph simple conics, including:</td>
<td>31</td>
<td>Trigonometry 4.0, 8.0, and 19.0; Math Analysis 6.0 and 7.0</td>
<td>G-GPE 1, 2, and 3</td>
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<tr>
<td>• a circle given its equation (centered anywhere)</td>
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<tr>
<td>• an ellipse given its equation (centered at the origin)</td>
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<tr>
<td>• a hyperbola given its equation (centered at the origin)</td>
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<tr>
<td>Solve statistical problems:</td>
<td>32</td>
<td>Embedded in Algebra I and Algebra II standards</td>
<td>A-APR 5 S-CP 9</td>
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<tr>
<td>• Compute permutations using fundamental counting principle.</td>
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<td>• Compute combinations.</td>
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<tr>
<td>• Compute probabilities using combinations.</td>
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<tr>
<td>• Compute probabilities using permutations.</td>
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<td>• Expand binomial expressions using the binomial theorem.</td>
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<td>Compute the general term and sums of arithmetic series, finite geometric series, and infinite geometric series.</td>
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<td>Algebra II 22.0 and 23.0</td>
<td>F-BF 2</td>
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<tr>
<td>Competency Statement (Precalculus)</td>
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<td>National Common Core Standard</td>
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<tr>
<td>Perform matrix operations, including addition and multiplication, and calculate the determinants of 2x2 and 3x3 matrices.</td>
<td>1</td>
<td>Linear Algebra 1.0–12.0</td>
<td>N-VM 6 and 8</td>
</tr>
<tr>
<td>Describe different types of functions (polynomial, exponential, logarithmic, and trigonometric) using a table, a graph, an equation, and a verbal description.</td>
<td>2</td>
<td>Trigonometry 4.0, 8.0, and 19.0; Math Analysis 6.0 and 7.0</td>
<td>F-IF 1, 2, 4, 5, 7, 8a, and 9</td>
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<td>Graph piecewise-defined functions.</td>
<td>3</td>
<td>No corresponding standard</td>
<td>F-IF 7a, 7b, and 7c</td>
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<td>Find all real zeros (roots) of polynomial functions exactly using the rational root theorem.</td>
<td>4</td>
<td>No corresponding standard</td>
<td>A-APR 3</td>
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<tr>
<td>Find all the zeros (roots) of a polynomial function.</td>
<td>5</td>
<td>No corresponding standard</td>
<td>N-CN 7 and 8</td>
</tr>
<tr>
<td>Solve polynomial inequalities.</td>
<td>6</td>
<td>No corresponding standard</td>
<td>F-IF 7c A-APR 3</td>
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<tr>
<td>Solve rational inequalities.</td>
<td>7</td>
<td>No corresponding standard</td>
<td>No corresponding standard</td>
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<tr>
<td>Find the composition of two or more functions, each containing two or more operations (such as linear, quadratic, rational, cubic, square root, cube root, and trigonometric functions).</td>
<td>8</td>
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<td>F-BF 1, 3, and 4</td>
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<tr>
<td>Given a function, determine if an inverse function exists. If so: • Find the inverse of the function algebraically and graphically. • Identify the domain and range of the original and inverse functions.</td>
<td>9</td>
<td>Higher level complexity than Algebra II 11.0 and 24.0</td>
<td>F-BF 4 F-IF 5</td>
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<tr>
<td>Competency Statement (Precalculus)</td>
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<td>CA Content Standard</td>
<td>National Common Core Standard</td>
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<tr>
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<tr>
<td>Simplify expressions involving exponents.</td>
<td>10</td>
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<td>No corresponding standard</td>
</tr>
<tr>
<td>Expand logarithmic expressions.</td>
<td>11</td>
<td>No corresponding standard</td>
<td>No corresponding standard</td>
</tr>
<tr>
<td>Condense logarithmic expressions.</td>
<td>12</td>
<td>No corresponding standard</td>
<td>No corresponding standard</td>
</tr>
<tr>
<td>Solve multistep exponential equations.</td>
<td>13</td>
<td>Higher level complexity than Algebra II 12.0</td>
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<tr>
<td>Solve multistep logarithmic equations.</td>
<td>14</td>
<td>No corresponding standard</td>
<td>No corresponding standard</td>
</tr>
<tr>
<td>Sketch the graph of a conic, identifying center, vertices, foci, and asymptotes (if present).</td>
<td>15</td>
<td>Math Analysis 5.0, 5.1, and 5.2</td>
<td>G-GPE 1, 2, and 3</td>
</tr>
<tr>
<td>Graph trigonometric functions of the form ( y = A \cdot f(Bx + C) + k ).</td>
<td>16</td>
<td>Trigonometry 2.0, 4.0, 5.0, and 9.0</td>
<td>F-TF 5</td>
</tr>
<tr>
<td>Given the graph of a trigonometric function, write the equation (sine, cosine, tangent).</td>
<td>17</td>
<td>Trigonometry 4.0</td>
<td>F-IF 7 F-TF 5</td>
</tr>
<tr>
<td>Graph inverse trigonometric functions.</td>
<td>18</td>
<td>No corresponding standard</td>
<td>F-TF 6</td>
</tr>
<tr>
<td>Prove trigonometric identities.</td>
<td>19</td>
<td>Trigonometry 3.0, 3.1, 3.2, 10.0, and 11.0</td>
<td>F-TF 8, 9, and 10</td>
</tr>
<tr>
<td>Competency Statement (Precalculus)</td>
<td>ACCESS Competency Number</td>
<td>CA Content Standard</td>
<td>National Common Core Standard</td>
</tr>
<tr>
<td>-----------------------------------</td>
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</tr>
<tr>
<td>Solve trigonometry equations.</td>
<td>20</td>
<td>Trigonometry 19.0</td>
<td>F-TF 7</td>
</tr>
</tbody>
</table>
| Solve for all unknown parts of a given triangle:  
  • by using right triangle trigonometry  
  • by using the Law of Sines  
  • by using the Law of Cosines | 21                       | Trigonometry 12.0 and 13.0 | G-SRT 8–11                  |
| Convert between polar and rectangular coordinates. | 22                       | Math Analysis 1.0   | G-PCC 1 and 2 (CA add-on standard) |
| Graph equations written in polar form. | 23                       | Trigonometry 15.0 and 16.0 | G-PCC 1 and 2 (CA add-on standard) |
| Graph equations written in parametric form. | 24                       | Math Analysis 7.0   | G-PCC 3 (CA add-on standard) |
| Given the components of a two-dimensional vector, determine the magnitude and direction of the vector. | 25                       | Math Analysis 7.0   | N-VM 1, 2, 3, and 4 |
| Perform the vector operations of addition, subtraction, and scalar multiplication. | 26                       | Linear Algebra 4.0, 5.0, and 7.0 | N-VM 4 and 5 |
| Use factorial notation.           | 27                       | No corresponding standard | No corresponding standard |
| Expand binomial expressions.      | 28                       | Higher level complexity than Algebra II 20.0 | A-APR 5 |
## Mapping ACCESS (Non-STEM) Statistics Competencies to Standards

<table>
<thead>
<tr>
<th>Competency Statement (Non-STEM Statistics)</th>
<th>ACCESS Competency Number</th>
<th>CA Content Standard</th>
<th>National Common Core Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine whether a variable is categorical or quantitative, given a situation.</td>
<td>1</td>
<td>Not explicitly stated, but implied in 8.0</td>
<td>Not explicitly stated, but implied in S-ID</td>
</tr>
<tr>
<td>Identify the sampling method used (random, systematic, convenience, stratified, cluster, or voluntary response), given a scenario in which data were collected from a population.</td>
<td>2</td>
<td>No corresponding standard</td>
<td>Not explicitly stated, but implied in S-ID 3</td>
</tr>
<tr>
<td>Identify the basic terminology of experimental design used in a given scenario.</td>
<td>3</td>
<td>No corresponding standard</td>
<td>Not explicitly stated, but implied in S-ID</td>
</tr>
<tr>
<td>Identify biases in experimental designs or in the creation of a sample.</td>
<td>4</td>
<td>No corresponding standard</td>
<td>S-IC 3</td>
</tr>
<tr>
<td>Determine an appropriate experimental design for a given scenario.</td>
<td>5</td>
<td>No corresponding standard</td>
<td>S-IC 3</td>
</tr>
<tr>
<td>Construct and describe an appropriate visual display given a data set (histogram, bar graph, stem plot, scatter plot, box and whisker plot, and pie chart).</td>
<td>6</td>
<td>8.0</td>
<td>S-ID 1 and 6</td>
</tr>
<tr>
<td>Identify which measure of the center (mean, median, or mode) is most appropriate for a given situation and what the relationship between the mean, median, and mode indicates about the distribution of the data.</td>
<td>7</td>
<td>6.0</td>
<td>S-ID 2</td>
</tr>
<tr>
<td>Determine standard deviation from multiple representations of data (table, graph, and word problems) and explain standard deviation in the context of a given situation.</td>
<td>8</td>
<td>5.0 and 7.0</td>
<td>S-ID 2</td>
</tr>
<tr>
<td>Competency Statement (Non-STEM Statistics)</td>
<td>ACCESS Competency Number</td>
<td>CA Content Standard</td>
<td>National Common Core Standard</td>
</tr>
<tr>
<td>----------------------------------------------------------------------------------------------------------</td>
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</tr>
<tr>
<td>Use the normal distribution to solve for the probability of an event or solve for the boundaries for a particular event.</td>
<td>9</td>
<td>4.0</td>
<td>S-ID 4</td>
</tr>
<tr>
<td>Determine the size of a sample space using counting principles, permutations, and combinations.</td>
<td>10</td>
<td>1.0 and 2.0</td>
<td>S-CP 9</td>
</tr>
<tr>
<td>Compute basic probability of independent events and solve problems involving the binomial distribution:</td>
<td>11</td>
<td>1.0 and 4.0</td>
<td>Not explicitly stated, but implied in S-MD</td>
</tr>
<tr>
<td>• Determine situations that are Bernoulli trials.</td>
<td></td>
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<tr>
<td>• Calculate the probability of an event (success) and its complementary probability (failure).</td>
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<td></td>
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</tr>
<tr>
<td>• Discern the difference between geometric probability and binomial probability of Bernoulli trials.</td>
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</tr>
<tr>
<td>• Apply the formula for binomial probability for $x$ successes in $n$ trials $P(x) = \binom{n}{x}p^xq^{n-x}$.</td>
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</tr>
<tr>
<td>• Use the normal model $N(\mu, \sigma)$, where $\mu = np$ and $\sigma = \sqrt{npq}$ to approximate a binomial probability when the number of trials and desired successes is inordinately large.</td>
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<tr>
<td>Apply the Central Limit Theorem to sampling distributions.</td>
<td>12</td>
<td>No corresponding standard</td>
<td>Not explicitly stated, but implied in S-IC</td>
</tr>
<tr>
<td>Estimate population parameters using confidence intervals for means and proportions.</td>
<td>13</td>
<td>No corresponding standard</td>
<td>S-IC 4</td>
</tr>
<tr>
<td>Competency Statement (Non-STEM Statistics)</td>
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<tr>
<td>Perform appropriate hypothesis test (state the hypotheses, determine the significance level, check conditions/criteria, calculate the test statistic, determine the p-value, make a decision and interpret it in the context of the problem) for a given situation.</td>
<td>14</td>
<td>No corresponding standard</td>
<td>Not explicitly stated, but implied in S-IC 4 and 5</td>
</tr>
<tr>
<td>Explain the relationship between parameters and statistics.</td>
<td>15</td>
<td>No corresponding standard</td>
<td>S-IC 1</td>
</tr>
<tr>
<td>Given a p-value, interpret its meaning in the context of the variables of a problem:</td>
<td>16</td>
<td>No corresponding standard</td>
<td>Not explicitly stated, but implied in S-IC 4 and 5</td>
</tr>
<tr>
<td>• Define the p-value.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>• Identify the appropriate null and alternative hypotheses, and use the proper notation of $H_0$ and $H_1$.</td>
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<tr>
<td>• Interpret a given p-value in the context of a hypothesis testing situation as it relates to the rejection of or failure to reject $H_0$.</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Find and apply the equation of the regression line when linear regression is appropriate:</td>
<td>17</td>
<td>No corresponding standard</td>
<td>S-ID 6, 7, and 8</td>
</tr>
<tr>
<td>• Determine when linear regression is appropriate for a set of bivariate data.</td>
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<tr>
<td>• Find the linear correlation coefficient $r$ and the coefficient of determination $r^2$.</td>
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<tr>
<td>• Use the regression line to make appropriate predictions.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>• Interpret $r$, $r^2$, and slope in the context of a given situation.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>